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Comparision of methods and software tools for availability assessment of production systems

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Problem Description

Availability and production availability are two central concepts when analysing if a production system is dependable and optimized for profit. Such analyses are important when decisions are to be made regarding implementation of new facilities or modifications on existing facilities. Assesment of production availability can be done in several ways, through classical statistical methods or through simulations by use of software tools.

All availability predictions involve uncertainty, regardless of method for assessment. Estimation of the uncertainty related to the results, will give the results a more useful value for the decision makers.

There exists several computer programs designed for availability or production availability assesment. Two of these are MIRIAM Regina, distributed by CognIt, and Relex Architect, distributed by Relex Scandinavia AB. There are also a number of statistical methods available, including renewal and quasi-renewal processes.

The candidate shall
- Compare the functionality, methods and results of the two computer programs MIRIAM Regina and Relex,
- Consider the uncertainty of the results from the two programs and compare these,
- Compare the results obtained by use of the computer programs with results obtained through other statistical methods.

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Abstract

This thesis presents and considers several different methods for computation of availability and production availability for production system. It is assumed that the system handles flow of a fluid. The thesis presents two software programs for computation of reliability measures, MIRIAM Regina and Relex Reliability Studio, and several analytical methods, among them one especially adapted to computation of production availability. For the methods not able to compute production availability, a method is presented which makes it possible to estimate production availability from computation of availability.

Among the methods, Relex and three of the analytical computation methods are made to compute availability of the system. The analytical methods considered are standard availability computation based on the structure function of the system and the definitions of availability and computation based on renewal and quasi renewal processes. Relex makes it possible to compute availability both by simulation and, if the system is simple enough, by analytical methods. The usefulness of the analytical methods is to an extent limited by the assumptions laid on the system. Relex makes it possible to take into account more features one would expect to have in a real life system, but for analytical methods to be employed in the computations, the system must be quite simple.

Two methods especially made for computing production availability are presented. These are the software program MIRIAM Regina, which combines a sophisticated flow algorithm with Monte Carlo simulation, and a method based on using Markov chains for computing the probability distribution for flow through subsystems of the system under consideration, and then employing simple merging rules to compute the flow through the entire system. These methods are both very flexible, and makes it possible to take into account many different aspects of a real life system.

The most important source of uncertainty in the results form a computation, lies in the relation between the real life system and the model system computations are made on. A model system will always be significantly simplified. When choosing a computation method and interpreting results, it is important to keep in mind all assumptions made regarding the system, both explicitly when making the model, and implicit in the computation method. Another source of uncertainty is uncertainty in the input data. A method for propagation of uncertainty through computations is presented and employed on some of the methods. For simulation, one will in addition have the uncertainty due to simulation being an way of making a statistical sample. The size of the sample, given by the number of simulation iterations done, will decide the accuracy of the result.

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Chapter 1

Introduction

This thesis concerns different methods for calculating availability and production availability for production systems. Both analytical methods and softwares for simulation will be considered.

The system type considered in this thesis is a production system where a flow of some kind enters the system at one point, goes through some process and leaves the system at another point. The starting point for this thesis was considerations for production systems in the petroleum industries, with liquid flows of oil, gas or water. In many cases, similar considerations may be used for other kinds of flow, for example parts being produced in a factory. What kind of flow goes through the system is not considered in this thesis, but the underlying assumption is that of a liquid flow. In the whole thesis, the calculated production availabilities are calculated as percent of full flow.

The focus of the thesis is on the presentation and comparison of the different methods for computation of availability and production availability. The background for the problem formulation is the fact that several different softwares for availability and production availability calculations exist and are employed in the petroleum industries. It is of interest to be able to compare results obtained from different softwares, and in some cases to be able to compute measurements which are not automatically computed by the software.

The softwares considered in this thesis are MIRIAM Regina, a software distributed by CognIT and widely used in Norway, and the software package Relex Reliability Studio 2007, distributed by Relex Scandinavia AB. In the thesis, the softwares will be referd to as respectively MIRIAM and Relex. The experience of the author of this thesis with both these softwares is limited, and getting to know especially Relex, has been a part of the task in the preparation of this thesis. The presentation of the softwares is as far as possible based on documentation from the distributors of the software.

In addition to the softwares, several analytical methods for computing availability and production availability are presented, as it may be of interest to compare the results of analytical computations with the results from the softwares, which are primarily based on simulation. A part of the thesis is also a method for estimating production availability from computations of availability. This is of interest primarily in the case when one does not have access to a software or method meant for computing production availability. In some cases, it might be easier to apply this method instead of employing time and effort in implementing the system in another software or using another method. In such cases it must be taken into consideration whether the estimates are sufficiently accurate for their uses.

1.1 Abbreviations

DFR	Decreasing Failure Rate
DFRA	Decreasing Failure Rate in Average
FMECA	Failure Modes, Effects and Criticality Analysis
IFR	Increasing Failure Rate
IFRA	Increasing Failure Rate in Average
iid	independent, identically distributed
LCL	Lower confidence limit
MDT	Mean Downtime
MooN	M out of N (components)
MTBF	Mean time between failures
MTTF	Mean Time To Failure
MTTR	Mean Time To Repair
NBU	New Better than Used
NBUE	New Better than Used in Expectation
NWU	New Worse than Used
NWUE	New Worse than Used in Expectation
RBD	Reliability block diagram
UCL	Upper confidence limit

1.2 Thanks

The problem formulation for this thesis was formulated in cooperation with Safetec Nordic A/S, and all software used during the process was made available by them.

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Chapter 2

Modeling of repairable systems

This thesis focuses on repairable systems, as availability primarily is a relevant measure for systems subject to downtime and repair. If a system cannot be repaired, only the reliability of the system is relevant unless there are other causes for downtime of the system which can not be quantified in advance. If the amount of downtime is known, the calculations are very simple.

2.1 Definitions and concepts

2.1.1 Measures for availability

There are several different concepts that are covered by the term availability. The basic definition used in the standard NORSOK Z-016 [16] is

> The ability of an item to be in a state to perform a required function under given conditions at a given instant of time or during a given time interval, assuming that the required external resources are provided. This ability is expressed as the proportion of time(s) the item is in the functioning state.
>
> Note 1: This ability depends on the combined aspects of the reliability, the maintainability and the maintenance supportability.
>
> Note 2: Required external resources, other than maintenance resources do not affect the availability of the item.

The basic mathematical definition of availability is [11]

$$A(t) = P(X(t) = 1) \tag{2.1}$$

where $X(t)$ is the structure function of the system in question. The structure function is such that

$$X(t) = \begin{cases} 1 & \text{if the system is functioning} \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

This expression for the availability gives the availability at a specific time. However, in many cases it will not be possible to determine this function.

Three other ways of calculating availability are of interest. These are

- interval or mission availability
- long run average availability
- limiting availability

The interval or mission availability is the average availability over a given interval of time, often adjusted to the mission time for the system being considered. This is calculated as

$$A_{\mathrm{av}}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} A(t)\mathrm{d}t \tag{2.3}$$

and can be regarded as the average amount of time the system is functioning during the interval in question, [11].

The long run average availability is the limit of the interval availability when the lower time limit is 0 and the upper time limit goes to infinity, that is

$$A_{\mathrm{av}} = \lim_{t \to \infty} \frac{1}{t} \int_{0}^{t} A(t)\mathrm{d}t \tag{2.4}$$

In some cases the availability of a system will reach a steady state after the system has been in operation for a time. In those cases the limiting availability is of use. It is defined as

$$A = \lim_{t \to \infty} A(t) \tag{2.5}$$

if the limit exists. All these definitions can be found in [11]. The limiting availability is of interest as for many systems, the availability quickly converges toward the limiting availability.

Availability measures the up time of the system as a whole. If any flow at all goes through the system, the system will be available. However, as flow through the system may vary significantly while the system is available, other measures are needed to accurately estimate the produced amount.

2.1.2 Production availability

Production availability is a concept related to availability. It is defined in [16] as

> The ratio of production to planned production, or any other reference level, over a specified period of time.
>
> Note 1: This measure is used in connection with analysis of delimited systems without compensating elements such as substitution from other producers and downstream buffer storage. Battery limits need to be defined in each case.
>
> Note 2: The term injection availability may be used meaning the ratio of injection volume to planned injection volume.

Expressed mathematically, this is

$$A_{\text{prod}} = \frac{P_{\text{produced}}}{P_{\text{reference}}} \tag{2.6}$$

Where A_{prod} denotes the production availability.

Production availability is found by looking at the availability of the system together with the capacity of the stages in the system. The estimated produced amount is then compared to the reference level, which often is the wanted or ideal production.

2.1.3 Other measures

Possible measures for the performance of the system with regard to meeting standards for production are operational availability and deliverability. The concepts are defined below, but neither are further considered in this thesis.

Operational availability is defined in [11] as

> The mean proportion of a mission period the item is able to perform its intended function.

It is found by estimating the mean planned and unplanned downtime of a system during its mission time.

Deliverability is defined in [16] as

> The ratio of deliveries to planned deliveries over a specified period of time, when the effect of compensating elements such as substitution from other producers and downstream buffer storage is included.

Deliverability is closely related to production availability, but concerns the amount which can be delivered and not the amount which can be produced, and so does not necessarily depend only on one production system. Deliverability is of concern when an amount is to be delivered to a customer, and is important for good customer relations.

2.1.4 Modeling of repair

When availability is to be calculated, repair must be modeled. The most common method is to assume that repair is perfect, that is, a component is as good as new after it is repaired. This is however not an assumption that fits well with the real world. In reality, a component can be close to as good as new, as bad as it was immediately before the failure and resulting repair, or anywhere in between. A component can even be in worse state after a repair, if mistakes were done by the person

In some cases, both repair and replacement of components can be done, and different assumptions can be assigned to the different maintenance actions.

Imperfect repair can be modeled in different ways. The most common is the (p, q) rule of imperfect repair.

The (p, q)-rule for imperfect repair

This method of modeling imperfect repair is easy to implement. When a repair is performed, it is assumed to be perfect with probability p and bad as old with probability q. If is is bad as old, the next time to failure is drawn from the tail of the probability distribution.

The method can be used for modeling repair with an arbitrary improvement of the component. The expected improvement from a repair will be somewhere between bad as old and good as new, depending on the value chosen for p.

For the exponential distribution, imperfect repair cannot be modeled, as this distribution is without memory, that is, for the exponential distribution,

$$P(X > x | X > x_0) = P(X > x) \tag{2.7}$$

where $0 \leq x_0 < x$. Thus, drawing a number from the tail of the distribution is the same as drawing from the distribution as a whole. This fact also shows as the exponential distribution has constant failure rate, see [11]

2.2 Uncertainty in the results

2.2.1 Model selection

Perhaps the largest error source in modeling of systems is the assumptions made when developing a model for the system under consideration. When creating a mathematical model of a real life system, simplifications must be made. In many cases it is impossible to calculate results unless one makes drastic simplifications. All such simplifications enlarges the divergence between the model and the real life system, and must be considered with the utmost care. When interpreting results, the assumptions made must be considered.

In [4], 18 factors which are often ignored in models of repairable systems are listed. Some of these factors can be put into categories as given below. Other factors are not mentioned here, as they primarily concern reliability calculations. For example, there is little meaning in assuming negligeble repair time when availability is to be calculated.

Independence of components

Perhaps the most common and most simplifying assumption made, is that of independence of components. To a certain extent, this is a plausible assumption, as each component in a system in many ways work on its own. On the other hand, there are many circumstances which may cause simultaneous failure of many components in a system, when several components depend on the same supporting system or are subject to the same environmental stress. Examples can be a failure in the electrical system or a lightning strike. These effects, known as common cause failures, are not taken into account when components are assumed to be independent. Such failures may also cause normally redundant components to fail simultaneously. Also, failure of one component may cause other components to fail.

Perfect repair

It is very common to assume that repair restores the system to a good as new state, again an assumption which is seldom true in the real world. A repair is usually performed by a person, who may make mistakes. A repair on a real system may rend the system good as new, as bad as it was prior to the repair or anywhere in between. It may even be worse than before the repair, or even be broken, if mistakes were made.

A repair may be incomplete, may cause damage or improve the system beyond what was expected. New parts may have been damaged before they are put into use, and so the repair using such parts will not be perfect. Also, parts may be replaced prior to failure if other repairs are done which make additional repairs easy to do.

An important aspect is that a preventive maintenance action may influence the steady state results. If maintenance are a part of the model, using steady state results to do calculations on the system may not be a good approximation. It may also be that the steady state implies a reliability or availability which is lower than the required level for the system, and the system must then be overhauled before it reaches the steady state.

Human error not taken into account

Failures due to human error are very often ignored when system reliability and availability are considered. This is at least partly due to the difficulty of assessing the probability of such errors. Human error may influence the day to day operation of the system, but is more likely to occur during repair or maintenance, and especially in unusual operating conditions, such as a when dangerous faults occur. This may influence the extent of damage when a failure occurs, and through this the time to repair.

Assumptions on the distributions of time to failure

Assumptions mentioned in [4] as being not well considered, are some which are often made without much thought. For example, the components of a system are mostly assumed to have the same failure and repair distributions during the whole time the calculations are done. In reality, they may have different distributions under different operating conditions.

Another problem, not mentioned in [4], is the very common assumption of exponential lifetime for components. This assumption gives a constant failure rate, when in practice one will most often observe that a system deteriorates as it ages. The exponential distribution does however make a good assumption if outside causes are assumed to be the predominating cause of failure. The exponential distribution is very easy to work with. This is probably the cause of its widespread use.

Simulation makes it possible to handle at least some of the above factors, but as the amount of simplification is reduced, implementation may become difficult and simulation time may increase. It is therefore limited how close to the real world situation it is possible to come. However computations are done, it is important to keep track of the assumptions made, and state these plainly together with the results.

2.2.2 Propagation of uncertainty

Uncertainty in the input data is also a major concern in many cases, as input data often is based on either prior observations or statements from manufactureres of components. Such input data will be subject to uncertainty.

When uncertain data is put into a model, the results will be affected by these uncertainties. This is called propagation of uncertainty. There are several methods for propagating uncertainty in input data to the results. Some of these methods are presented in [10].

The uncertainty is expressed as the product of the stability and the variation, as argued in [10]. If y is the output value, and a function of the input value x, the sensitivity of the result can be measured as the derivative of y with respect to x evaluated for a baseline value x_0. The variance of the output can then be estimated as

$$\mathrm{Var}(y) \approx \left(\frac{\partial y}{\partial x}\right)^2_{x_0} \mathrm{Var}(x) \tag{2.8}$$

Both analytical methods and methods for simulated models exist. For analytical methods, one of the most common methods is Taylor series approximation.

Let $X = [x_i]_{i=1}^n$ be a vector of input values, and let $y = f(X)$. Let the baseline values be equal to the expectation of the input values, so $X^0 = [x_i^0]_{i=1}^n = \mathrm{E}(X)$ and $y^0 = f(X^0)$. Then the Taylor series expansion of the distance between the output and the expected output is

$$y - y^0 0 = \sum_{i=1}^{n}(x_i - x_i^0)\left[\frac{\partial y}{\partial x_i}\right]_{X^0} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}(x_i - x_i^0)(x_j - x_j^0)\left[\frac{\partial^2 y}{\partial x_i \partial x_j}\right]_{X^0} + \dots \tag{2.9}$$

Close to X^0 the higher powers of the series will be small. One can then use only two terms of the expansion and get a good approximation.

$$\begin{aligned} \mathrm{E}(y - y^0) &\approx \sum_{i=1}^{n}\mathrm{E}(x_i - x_i^0)\left[\frac{\partial y}{\partial x_i}\right]_{X^0} \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{E}[(x_i - x_i^0)(x_j - x_j^0)]\left[\frac{\partial^2 y}{\partial x_i \partial x_j}\right]_{X^0} \\ &= \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{Covar}(x_i, x_j)\left[\frac{\partial^2 y}{\partial x_i \partial x_j}\right]_{X^0} \end{aligned} \tag{2.10}$$

As x_i^0 is the expected value of x_i, the expected value of $x_i - x_i^0$ is 0. The expression $\left[\frac{\partial y}{\partial x_i}\right]_{X^0}$ denotes the value of the derivative when evaluated at X^0.

It is also common to use first order approximation, for which the result for the expected value of the deviation $y - y_0$ is approximately 0. One can then compute an approximation to the variance of $y - y_0$:

$$
\begin{aligned}
\mathrm{Var}(y) &= \mathrm{E}[(y - y^0)^2] \\
&\approx \mathrm{E}\left[\left(\sum_{i=1}^{n}(x_i - x_i^0)\left[\frac{\partial y}{\partial x_i}\right]_{X^0}\right)^2\right] \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{E}[(x_i - x_i^0)(x_j - x_j^0)]\left[\frac{\partial y}{\partial x_i}\right]_{X^0}\left[\frac{\partial y}{\partial x_j}\right]_{X^0} \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{Covar}(x_i, x_j)\left[\frac{\partial y}{\partial x_i}\right]_{X^0}\left[\frac{\partial y}{\partial x_j}\right]_{X^0} \\
&= \sum_{i=1}^{n}\mathrm{Var}(x_i)\left[\frac{\partial y}{\partial x_i}\right]_{X^0}^2 \\
&+ 2\sum_{i=1}^{n}\sum_{j=i+1}^{n}\mathrm{Covar}(x_i, x_j)\left[\frac{\partial y}{\partial x_i}\right]_{X^0}\left[\frac{\partial y}{\partial x_j}\right]_{X^0}
\end{aligned}
\tag{2.11}
$$

If the input values are assumed to be independent or uncorrelated, one is left with only the variance terms.

Methods for Monte Carlo simulation

When using Monte Carlo simulation, the expectation and the variance of the output can be estimated from the output values of the simulation runs, but it is important to do enough simulation runs. To compute how many simulation runs are needed to obtain a certain width of the confidence intervals computed, one can estimate the variance by doing a few simulation iterations and then use the formula for the confidence interval based on the central limit theorem to compute the needed number of simulations.

The central limit can be found in any basic book on statistics, for example [13]. The theorem states that the statistic

$$Z = \frac{\bar{y} - \mu}{\frac{\sigma}{\sqrt{n}}} \tag{2.12}$$

where $\bar{y}$ is the mean of a random sample from a population with mean value μ and variance σ^2 is distributed according to the standard normal distribution when $n \to \infty$. This can be used to compute a confidence interval for the mean. A $1 - \alpha\%$ confidence interval is then given by

$$\left[\bar{y} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{y} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right] \tag{2.13}$$

and the width of the confidence interval is found by subtracting the lower limit from the upper limit. This gives

$$\begin{aligned} W &= \bar{y} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} - \left(\bar{y} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right) \\ &= 2 z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \end{aligned} \tag{2.14}$$

For a given width W, the equation can be solved for n. The result is

$$n = 4 \left[\frac{z_{\frac{\alpha}{2}}}{W} \right]^2 \sigma^2 = 4 \left[\frac{z_{\frac{\alpha}{2}}}{W} \right]^2 \mathrm{Var}(y) \tag{2.15}$$

There are also methods for computing uncertainty importance for Monte Carlo models, but these need to be implemented in the simulation software, as it is necessary to know all input values, and these are drawn as the simulation progresses and usually not stored as this would require very much storage capacity.

One possible measure is the sample correlation. Let m be the number of runs, and let x_k be a single input value in run number k and y_k be the corresponding output. The correlation between these can then be computed as

$$U(x, y) = \frac{\sum_{k_1}^{m} (x_k - \bar{x})(y_k - \bar{y})}{\sqrt{\sum_{k=1}^{m} (x_k - \bar{x})^2 \times \sum_{k=1}^{m} (y_k - \bar{y})^2}} \tag{2.16}$$

This can be computed for each input value. This assumes that each simulation run has the same number of input values, which is not the case for many simulation methods. One could for example do the calculations for the first input drawn for each item being simulated. To do the calculations at all, it will be necessary to implement the method in the simulation methodology. If the method is not implemented when the simulation is done, it cannot be employed.

2.2.3 Uncertainty due to the procedure

In simulation, the error will depend on the number of simulations made. This is due to the fact that simulation is a way of creating a random sample. The size of the sample will be determined by the number of simulation iterations, and is important to the accuracy of the computations made using the data.

The number of simulations will most often depend on cost of simulation and available time. Assumptions for the model will of course also give rise to uncertainty.

For analytical methods the errors arising will depend on whether the assumptions for the methods and models are good enough and fit the real life situation.

The assumptions in an analytical model will be fashioned to come close to a real life system, but will to a large degree depend on what can be calculated. For large systems, analytical calculations can be prone to round off error.

Chapter 3

Methods for availability calculations

3.1 Calculation of availability

When it is assumed that the components of a system are independent, availability of the system can be calculated from the structure function. The structure function of a system is found by applying the simple rules for parallel and series structures. For a series structure, the system will fail if one of the components fail, so all components must be functioning for the system to function. The structure function of a series structure consisting of n components with

$$x_i = \begin{cases} 1 & \text{if the component is functioning} \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

is

$$X = \prod_{i=1}^{n} x_i \tag{3.2}$$

For a parallel structure, the system will function as long as one or more components are functioning. The structure function is given by

$$X = 1 - \prod_{i=1}^{n} (1 - x_i) \tag{3.3}$$

These equations can be combined when a system consists of both parallel and series structures. If the system is represented by a reliability block diagram, it will in most cases be possible to use these equations directly. For some structures, it will be easier to find the structure function by employing pivotal decomposition or other methods which enable division of the system into more easily manageable parts, see [11].

The structure function can also be found from the cut sets of a system. A cut set is a set of components so that if all components of the cut set are failed, the system has failed. For the system shown in figure 3.1, the cut sets are $\{1\}, \{2, 3, 4\}, \{5\}, \{6, 7\}$. The structure function can then be made by considering the components in the cut sets to be in parallel, and the cut sets themselves to be in series, as all cut sets must function, but only one component in each cut set is needed for the system to function.

Availability for the components can be calculated in many ways, the most common of which is to compute it as

$$a_i = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} \tag{3.4}$$

which gives the mean availability. However, many other methods may be used.

When analytical methods are used, it is assumed that the result is accurate. Usually, no confidence interval or similar measure is computed. The accuracy of the result will depend on the degree to which the assumptions of the method agree with the real world situation. This may lead to grave errors if not handled with care, as many assumptions made in analytical methods are completely implausible in the real world, as discussed previously. Another source of error in analytical calculations is inaccuracy in the input data. This has also been discussed in previous sections.

Most methods presented in this thesis assume independent components. If this assumption is to be avoided, possible methods are Markov chains and simulation. Later sections will present both of these options.

3.2 Calculation of production availability

If it is assumed that the system can either produce at full capacity or not produce at all, production availability can be found by multiplying the mission availability with the production capacity of the system and dividing it by planned production. In many cases, the problem will be far more complex, as the production capacity can be reduced by an accident, and the reduction in capacity can vary with which component or combination of components that are not functioning. The problem is the calculation of the flow through the network, and the calculation of amounts of time the system is in different states.

As analytical methods have limitations when production availability is to be calculated, simulation is often used. Next event Monte Carlo simulation is especially useful in this context, as it can be adapted to almost any system. For a complex system, the flow calculation will be a challenge, but there are algorithms for such problems, see for example [6] where maximum flow algorithms are presented. MIRIAM Regina is an example of a software combining next event Monte Carlo simulation with a flow algorithm for calculation of production availability. Simulation is more complex than analytical calculation if the simulation algorithm must be implemented before simulation can start. If a software with the simulation algorithm already implemented is available, simulation may be less complex than analytical calculations.

3.3 An analytical method for calculating production availability

In [9] an analytical method for calculating production availability is described. The method is based on finding the probability distribution for the flow through the system by for example using Markov chains to find the probability distribution for subsystems and then applying some simple rules for merging the results of the subsystems. The merging rules can be applied regardless of how the probability distribution for the subsystems was found.

If Markov chains are the preferred method, the calculation proceeds as follows: The system under consideration is divided into subsystems. For each subsystem, all possible states must be listed, and a Markov chain established, se

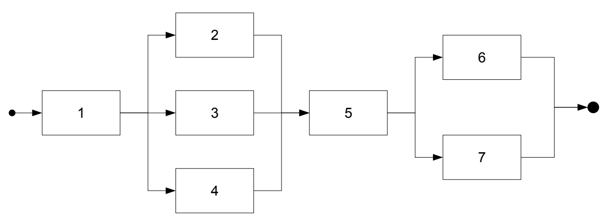

Figure 3.1: The example system

According to the article [9], one of the advantages of the analytical methods is ease of calculation, but the advantage of simulation is the possibility of incorporating complex maintenance and repair strategies in the model. The reason for using Markov modeling in the method proposed in the article is to make it possible to incorporate such strategies into the analytical

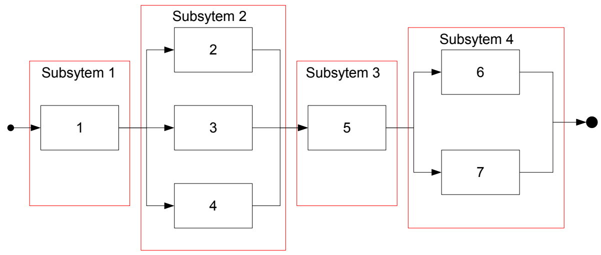

Figure 3.2: The example system with subsystems shown as red frames

model. Markov chains can be adapted to almost any degree of complexity, though the number of possible states for a system will increase rapidly. To an extent this can be countermanded by splitting the system up into more subsystems. The limitations for the method consists in the amount of work and computation time available.

The method consists of first dividing the system into subsystems and analyzing the subsystems to find the different states it can be in. Each state of the subsystem will be associated with a production volume decided by the capacities of the components that are functioning. Markov modeling is then used to establish a probability distribution for the production capacity. When this is done for all subsystems, a set of simple rules is used to merge the probability distribution for the different subsystems. The merging rules are as follows:

For two subsystems connected in series, with discrete probability distribution for the production volume: Let X be the volume produced by the first subsystem, and Y be the volume produced by the second subsystem, and let Z denote the produced volume of the merged subsystem. Then

$$P(Z = z) = P(X = z)P(Y \geq z) + P(Y = z)P(X > z) \qquad (3.5)$$

If the probability distributions are continuous, let X $f(x)$, Y $g(y)$ and Z $h(z)$. The rule is

$$h(z) = f(z) \int_z^{y_{\max}} g(y)\mathrm{d}y + g(z) \int_z^{x_{\max}} f(x)\mathrm{d}x \qquad (3.6)$$

For two subsystems connected in parallel, with discrete probability distributions, the rule is

$$P(Z = z) = P(X + Y = z) = \sum_{x,y;x+y=z} P(X = x)P(Y = y) \tag{3.7}$$

and with continuous probability distribution,

$$h(z) = \int_{x_{\min}}^{z} f(x)g(z - x)\mathrm{d}x \tag{3.8}$$

3.3.1 The method used on an example

The example system is shown in figure 3.1 above. The subsystem division is shown in figure 3.2, and data for the system is shown in table 3.1.

Component	**MTTF** (hours)	λ (hours^{-1})	**MTTR** (hours)	μ (hours^{-1})	**Availability**	**Capacity** %
1	500	0.002	3	0.33	0.994	100
2	600	0.00167	7	0.143	0.988	40
3	400	0.0025	4	0.25	0.99	60
4	700	0.00143	2	0.5	0.997	50
5	600	0.00167	9	0.11	0.985	100
6	500	0.002	7	0.143	0.986	50
7	600	0.00167	4	0.25	0.993	60

Table 3.1: Data for the system in the example

All subsystems in this example have a finite number of possible states, and the probability distributions for the flow are then discrete. For subsystem 1, which only contains one component, there are only two states, failed or in order. The probability distribution is thus easily determined. Either, the component is working, and 100% flow goes through the subsystem, or the component is failed and no flow can pass. The probability of 100% flow is the same as the probability of the component being in a functioning state. The states of the simple systems are given in table 3.2, and the transition diagram is shown in figure 3.4.

State No.	**Description**	**Capacity**
1	Component working	100
2	Component failed	0

Table 3.2: The possible states of subsystems number 1 and 3

Subsystem 2 has three components and a number of possible states, listed in table 3.3

Let λ_i and μ_i denote respectively the failure and repair frequencies of component number i. To compute the probability distribution, Markov

State No.	Description	Capacity
1	All components working	100
2	Comp. 2 and 3 working, 4 failed	100
3	Comp. 3 and 4 working, 2 failed	100
4	Comp. 2 and 4 working, 3 failed	90
5	Comp. 2 working, 3 and 4 failed	40
6	Comp. 3 working, 2 and 4 failed	60
7	Comp. 4 working, 2 and 3 failed	50
8	All failed	0

Table 3.3: The possible states of subsystem number 2

chains are employed. Basic Markov chain theory can be found for example in [11] and [12]. The transition diagram of the system is shown in figure 3.3. The transition matrix of the Markov chain is then

$$\begin{bmatrix} \alpha_1 & \lambda_3 & \lambda_1 & \lambda_2 & 0 & 0 & 0 & 0 \\ \mu_4 & \alpha_2 & 0 & 0 & \lambda_3 & \lambda_2 & 0 & 0 \\ \mu_2 & 0 & \alpha_3 & 0 & 0 & \lambda_4 & \lambda_3 & 0 \\ \mu_3 & 0 & 0 & \alpha_4 & \lambda_4 & 0 & \lambda_2 & 0 \\ 0 & \mu_3 & 0 & \mu_4 & \alpha_5 & 0 & 0 & \lambda_2 \\ 0 & \mu_2 & \mu_4 & 0 & 0 & \alpha_6 & 0 & \lambda_3 \\ 0 & 0 & \mu_3 & \mu_2 & 0 & 0 & \alpha_7 & \lambda_4 \\ 0 & 0 & 0 & 0 & \mu_2 & \mu_3 & \mu_4 & \alpha_8 \end{bmatrix} \tag{3.9}$$

Where

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{bmatrix} = \begin{bmatrix} -(\lambda_2 + \lambda_3 + \lambda_4) \\ -(\lambda_2 + \lambda_3 + \mu_4) \\ -(\lambda_3 + \lambda_4 + \mu_2) \\ -(\lambda_2 + \lambda_4 + \mu_3) \\ -(\mu_3 + \mu_4 + \lambda_2) \\ -(\mu_2 + \mu_4 + \lambda_3) \\ -(\mu_2 + \mu_3 + \lambda_4) \\ -(\mu_2 + \mu_3 + \mu_4) \end{bmatrix} \tag{3.10}$$

Subsystem number three again consists of only one component, and is treated just like subsystem one. Subsystem 4 consists of two components in parallel, and has four states, as described in table 3.4.

The transition diagram is shown in figure 3.5, and the transition matrix for subsystem 4 is

$$\begin{bmatrix} 1-(\lambda_6+\lambda_7) & \lambda_7 & \lambda_6 & 0 \\ \mu_7 & 1-(\lambda_6+\mu_7) & 0 & \lambda_6 \\ \mu_6 & 0 & 1-(\lambda_7+\mu_6) & \lambda_7 \\ 0 & \mu_6 & \mu_7 & 1-(\mu_6+\mu_7) \end{bmatrix} \tag{3.11}$$

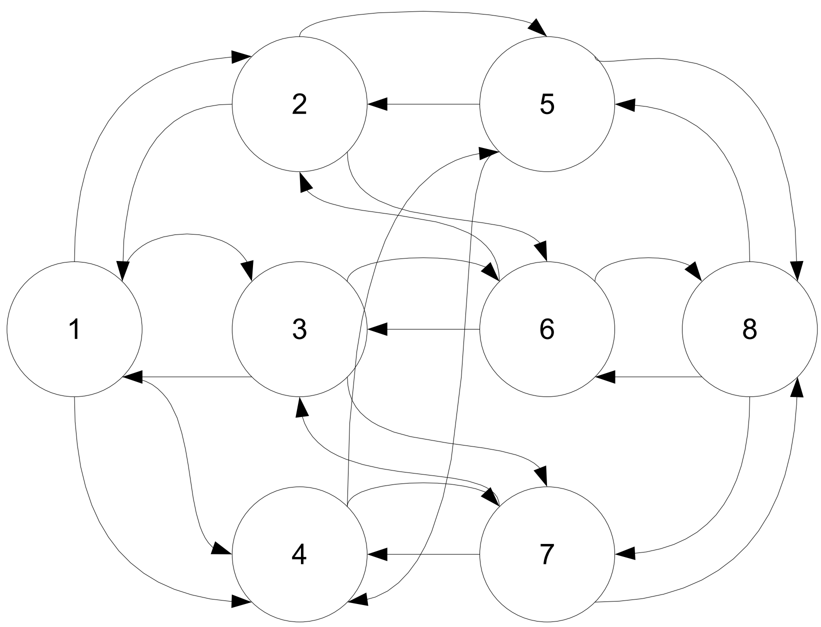

Figure 3.3: The transition diagram for subsystem 2

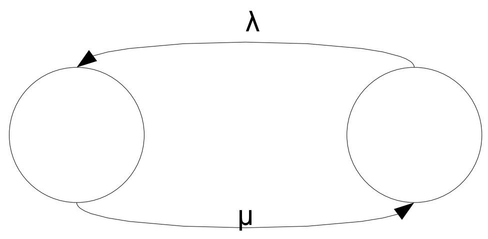

Figure 3.4: The transition diagram for subsystems 1 and 4

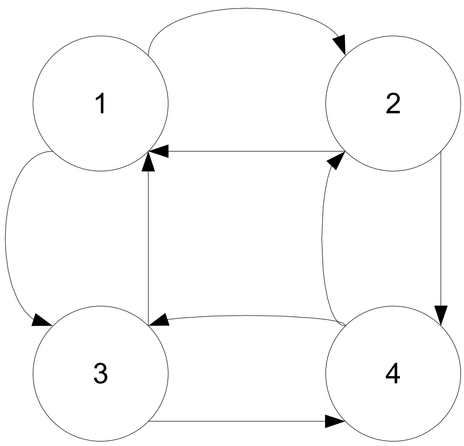

Figure 3.5: The transition diagram for subsystem 3

State No.	Description	Capacity
1	Both components working	100
2	Comp. 6 working, 7 failed	50
3	Comp. 7 working, 6 failed	60
4	Both components failed	0

Table 3.4: The possible states of subsystem number 4

To calculate the probability distribution for the different states, one must find the limiting probabilities for the states. As the chains are all irreducible and positive recurrent, the limiting probabilities exists and can be found from the equations

$$\begin{aligned} \pi_j &= \sum_i \pi_i P_{ij} \\ \sum_j \pi_j &= 1 \end{aligned} \tag{3.12}$$

where P_{ij} denotes the transition probability from state i to state j. These equations give rise to a system of linear equations, which can easily be solved by mathematical software equipped for matrix computations. The limiting probabilities can be interpreted as the amount of time the system spends in the respective states. For subsystem 4, the equation is

$$\begin{bmatrix} -(\lambda_6+\lambda_7) & \mu_7 & \mu_6 & 0 \\ \lambda_7 & -(\lambda_6+\mu_7) & 0 & \mu_6 \\ \lambda_6 & 0 & -(\lambda_7+\mu_6) & \mu_7 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.13}$$

When numbers are inserted and the equation solved (using the statistical software `R` which includes facilities for matrix computations), the solution is found to be

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} 9.7966 \cdot 10^{-1} \\ 6.5441 \cdot 10^{-3} \\ 1.3702 \cdot 10^{-2} \\ 9.1527 \cdot 10^{-5} \end{bmatrix} \tag{3.14}$$

For the other subsystems, the calculations are done in exactly the same manner. Subsystem 1, consisting of only one component has the limiting probabilities

$$\begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix} = \begin{bmatrix} 0.9940 \\ 0.0060 \end{bmatrix} \tag{3.15}$$

Subsystem 2 gives the limiting probabilities

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \\ \pi_6 \\ \pi_7 \\ \pi_8 \end{bmatrix} = \begin{bmatrix} 9.758788 \cdot 10^{-1} \\ 2.791013 \cdot 10^{-3} \\ 1.139663 \cdot 10^{-2} \\ 9.758788 \cdot 10^{-3} \\ 2.791013 \cdot 10^{-5} \\ 3.259435 \cdot 10^{-5} \\ 1.139663 \cdot 10^{-4} \\ 3.259435 \cdot 10^{-7} \end{bmatrix} \tag{3.16}$$

and for subsystem 3, the result is

$$\begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix} = \begin{bmatrix} 0.9850 \\ 0.0150 \end{bmatrix} \tag{3.17}$$

With the results above, the probability distributions for the flow through each subsystem is determined. Now the subsystems must be merged, using the rules stated in section 3.3 above. First, subsystems one and two are merged. The possible amounts of flow for subsystem 1 are 0% and 100%. Subsystem two has the possibilities 0%, 40%, 50%, 60%, 90% and 100%. (Subsystem two in reality has larger capacity, but will never recieve flow greater than 100%, so this is ignored). The subsystems are connected in series and have a finite number of possible amounts of flow, so the probability distribution of the flow is determined by equation (3.5). X is the flow through the first subsystem, Y is the flow through the second.

$$\begin{aligned} P(Z=0) &= P(X=0)P(Y \geq 0) + P(Y=0)P(X>0) \\ &= 0.0060 \cdot 1 + 3.2594 \cdot 10^{-7} \cdot 0.9940 \\ &= 6.0003 \cdot 10^{-3} \end{aligned} \tag{3.18}$$

$$\begin{aligned} P(Z=40) &= P(X=40)P(Y \geq 40) + P(Y=40)P(X>40) \\ &= 0 + 2.7910 \cdot 10^{-5} \cdot 0.9940 \\ &= 2.7743 \cdot 10^{-5} \end{aligned} \tag{3.19}$$

$$\begin{aligned} P(Z=50) &= P(X=50)P(Y \geq 50) + P(Y=50)P(X>50) \\ &= 0 + 1.1397 \cdot 10^{-4} \cdot 0.9940 \\ &= 1.1329 \cdot 10^{-4} \end{aligned} \tag{3.20}$$

$$\begin{aligned} P(Z=60) &= P(X=60)P(Y \geq 60) + P(Y=60)P(X>60) \\ &= 0 + 3.2594 \cdot 10^{-5} \cdot 0.9940 \\ &= 3.2398 \cdot 10^{-5} \end{aligned} \tag{3.21}$$

$$
\begin{aligned}
P(Z=90) &= P(X=90)P(Y \geq 90) + P(Y=90)P(X>90) \\
&= 0 + 9.7588 \cdot 10^{-3} \cdot 0.9940 \\
&= 9.7002 \cdot 10^{-3}
\end{aligned}
\tag{3.22}
$$

$$
\begin{aligned}
P(Z=100) &= P(X=100)P(Y \geq 100) + P(Y=100)P(X>100) \\
&= 0.994 \cdot (9.7588 \cdot 10^{-1} + 2.7910 \cdot 10^{-3} + 1.1397 \cdot 10^{-2}) + 0 \\
&= 0.98413
\end{aligned}
\tag{3.23}
$$

To sum up, the probability distribution for the different amounts of flow for the merged subsystem is given in table 3.5p, the probability distribution for the different amounts of flow for the merged subsystem is given in table 3.5 Next, subsystems 3 and 4 are merged. The process is as described above,

Amount of flow	**Probability**
0	$6.0003 \cdot 10^{-3}$
40	$2.7743 \cdot 10^{-5}$
50	$1.1329 \cdot 10^{-4}$
60	$3.2398 \cdot 10^{-5}$
90	$9.7002 \cdot 10^{-3}$
100	0.98413

Table 3.5: The probatility distribution for the flow through the merging of subsystem 1 and 2

and the result is given in table 3.6. At last, the two merged systems are

Amount of flow	**Probability**
0	0.01509
50	0.006446
60	0.01350
100	0.9650

Table 3.6: The probability distribution for the flow through the merging of subsystem 3 and 4

merged, and the result for the whole system is obtained. It is given in table 3.7.

The production availability of the system then becomes

$$
\begin{aligned}
A_{\text{production}} =& 40 \cdot 2.7324 \cdot 10^{-5} + 50 \cdot 6.5180 \cdot 10^{-3} + 60 \cdot 1.3448 \cdot 10^{-2} + 90 \cdot 9.3607 \cdot 10^{-3} \\
&+ 100 \cdot 0.94969 = \underline{\underline{96.94533}}
\end{aligned}
\tag{3.24}
$$

Amount of flow	**Probability**
0	$2.1000 \cdot 10^{-2}$
40	$2.7324 \cdot 10^{-5}$
50	$6.5180 \cdot 10^{-3}$
60	$1.3448 \cdot 10^{-2}$
90	$9.3607 \cdot 10^{-3}$
100	0.94969

Table 3.7: The probability distribution for the flow through the whole system.

If one compares this result to the result obtained in the section above, the results are quite similar. The availability of the system can be computed as the amount of time the flow is larger than 0, that is

$$1 - 2.1 \cdot 10^{-2} = 0.979 \tag{3.25}$$

3.4 %availability

An alternative and less accurate approach to estimating the produced amount from a system is to compute the amount of time when a given percentage of the demanded flow is available. This gives an idea of how much difference there is between the availability of a system and the production availability.

For systems with parallel structures, different configurations are possible. If a system has three parallel streams, one, two or all three may have to work for the system to be functioning. Generally, such configurations are denoted by the expression $M\text{oo}N$, interpreted as M out of N components must function for the system to function. A system with tree components in parallel can thus have the configurations 1oo3, 2oo3 or 3oo3. Availability for the system can be computed for all such configurations.

If considering the system of the example above, shown in figure 3.1, with data given in table 3.1, it is easily seen that to be sure of having 100 percent flow passing through the system, all components must be operational. 90 percent flow is ensured if the first parallel has configuration 2oo3 and the second configuration 3oo3. If one has configuration 1oo3 and 1oo2, 40 percent flow is ensured, but 50 or 60 percent is more likely. These different availabilities can be computed by applying the structure function of the system. The structure function for the system with configuration 3oo3 and 2oo2 respectively for subsystem 2 and 4, is given by

$$A = a_1 a_2 a_3 a_4 a_5 a_6 a_7 \tag{3.26}$$

as this is the same as requiring all components to be working.

For configuration 2oo3 for subsystem 2 and 2oo2 for subsystem 4, the availability is given by

$$A = a_1(1-(1-a_2a_3)(1-a_2a_4)(1-a_3a_4))a_5a_6a_7 \tag{3.27}$$

and for configurations 2oo3 and 1oo2, the equation is

$$A = a_1(1-(1-a_2a_3)(1-a_2a_4)(1-a_3a_4))a_5(1-(1-a_6)(1-a_7)) \tag{3.28}$$

When numbers are inserted, the probabilities are found to be as given in table 3.8. It is clear that 100% flow is available most of the time, but as seen, approximatly 3.5% of the time when the system is available, it does not produce full flow.

Percent available	**Availability**
40	0.97899
50	0.97898989
90	0.95862
100	0.93484

Table 3.8: The percent availabilities for the example system

3.5 Computing production availability when availability is known

In this thesis, it is of interest to compare results obtained from different methods. One of these methods, the reliability software MIRIAM Regina, calculates production availability. Another method, the software Relex reliability studio, computes availability. It will also be of interest to compare the results with results obtained from analytical methods. It is thus of interest to be able to compare production availability and availability.

For a simple series structure, the production availability can easily be calculated from the availability by multiplying the computed availability with the capacity of the structure. The capacity of a series structure is decided by the component with the lowest capacity. For more complex structures, the problem of calculating the production availability quickly gets far more involved.

When computing availability, one assumes that the system is either working or not working. In computing production availability, degraded states of the system are considered if they are present. One can however compare the amount of time with no production with the general unavailability calculation. These should be comparable, as no flow through the system is the same as the system being unavailable, provided that there is no way there can be flow going through the system when availability calculations consider the system to be failed.

If a more accurate calculation is needed, the system must be considered in more detail. Difficulties arise when there are several alternate routes for the production flow with different capacities. It is then necessary to determine the average amount of time when each of these routes are being used.

When computing the availability of the system with configuration 1oo3, the result is the amount of time when 0, 1 or 2 streams are failed, that is, 1, 2 or 3 streams are working. Using configuration 2oo3 gives the time when 2 or 3 stream are working, and configuration 3oo3 gives the time when 0 streams have failed, and consequently all streams are working. By calculating all these availabilities and looking at the differences between them, one can find the amounts of time when exactly 1, 2 or 3 streams are working.

Assuming that the ratio of time for these states is constant whether the system is working or not, one can find the probability of 1, 2 or 3 streams working given that the system is working. This is a natural assumption as long as the components all have time measured as either calendar time or running time.

For three components in parallel, denoted 1, 2 and 3, with availability a_1, a_2 and a_3, the relevant expressions can be found by first determining the structure function for each of the components and then inserting the availability of the components instead of the state variables x_1, x_2 and x_3. The structure functions are:

Configuration 1oo3:

$$\begin{aligned} X(t) &= 1 - (1 - x_1)(1 - x_2)(1 - x_3) \\ &= 1 - (1 - x_1 - x_2 + x_1x_2)(1 - x_3) \\ &= 1 - (1 - x_1 - x_2 + x_1x_2 - x_3 + x_1x_3 + x_2x_3 - x_1x_2x_3) \\ &= x_1 + x_2 + x_3 - x_1x_2 - x_2x_3 - x_1x_3 + x_1x_2x_3 \end{aligned} \tag{3.29}$$

Configuration 2oo3:

$$\begin{aligned} X(t) &= 1 - (1 - x_1x_2)(1 - x_2x_3)(1 - x_1x_3) \\ &= 1 - (1 - x_1x_2 - x_2x_3 + x_1x_2^2x_3)(1 - x_1x_3) \\ &= 1 - (1 - x_1x_3 - x_1x_2 + x_1^2x_2x_3 - x_2x_3 + x_1x_2x_3^2 + x_1x_2^2x_3 - x_1^2x_2^2x_3^2) \\ &= x_1x_2 + x_2x_3 + x_1x_3 - x_1^2x_2x_3 - x_1x_2^2x_3 - x_1x_2x_3^2 + x_1^2x_2^2x_3^2 \end{aligned} \tag{3.30}$$

Configuration 3oo3 equals a series structure:

$$X(t) = x_1x_2x_3 \tag{3.31}$$

3.5.1 Example

An easy example of the method described above. The system consists of seven components, for which data is given in table 3.1. A diagram showing the flow through the system is shown in figure 3.1. The capacities are given in percent of required flow. It is assumed that all components are independent. Further, it is assumed that the steady state availabilities can be employed.

The steady state availability of the system is easily calculated from the structure function together with the definition of the steady state availability. If the second subsystem is assumed to have configuration 1oo3 and the fourth subsystem has configuration 1oo2, the structure function is

$$X = x_1(1-(1-x_2)(1-x_3)(1-x_4))x_5(1-(1-x_6)(1-x_7)) \qquad (3.32)$$

When the steady state availabilities given in table 3.1 are put into this equation, the result is $A = 0.97899$.

To compute the production availability of the system, the system is first decomposed into subsystems which are in series with each other. Thus, there are 4 subsystems, the first consisting of only component number 1, the second of the three components 2, 3 and 4 which are in parallel with each other, the third being only component number 5 and the last consisting of components number 6 and 7, which again are in parallel. This is shown in figure 3.2. When decomposing into such subsystems, it is important that each component in the system only is part of one subsystem, as one otherwise will have dependencies between the subsystems. In this case this is no problem.

For the first subsystem, calculating production availability is easy, as the component will either let through 100% of the wanted production, or none.

For the subsystem consisting of three components in parallel, the situation is that if all three components function, there is superfluous capacity. If only two streams are available, one may or may not have the demanded capacity, depending on which streams are available. If only one stream is available, how much capacity is available will also depend on which stream is functioning.

To calculate the availability of the streams, first calculate the availability if the system has configuration 1oo3. This is given by

$$A_{1oo3} = 1-(1-a_2)(1-a_3)(1-a_4) = 0.99999964 \qquad (3.33)$$

For configuration 2oo3, the equation is

$$A_{2oo3} = 1-(1-a_2a_3)(1-a_3a_4)(1-a_2a_4) = 0.999995754 \qquad (3.34)$$

and for configuration 3oo3, which equals having a series system, as all components must function for the system to function,

$$A_{3oo3} = a_2 a_3 a_4 = 0.97518564 \tag{3.35}$$

By looking at the differences between these, one can find the amount of time when 1, 2 or 3 streams are available. Configuration 1oo3 gives the amount of time when 1, 2 or 3 streams are available, configuration 2oo3 gives the availability of 2 or 3 streams and configuration 3oo3 gives the availability of 3 streams. Thus 3 streams are available

$$A_{3oo3} = 0.975 \tag{3.36}$$

that is, 97.5% of the time. Two streams have avilability

$$A_{2oo3} - A_{3oo3} = 0.024996 \tag{3.37}$$

that is they are available 2.4996% of the time, and only one stream is available

$$A_{1oo3} - A_{2oo3} = 3.883 \cdot 10^{-6} \tag{3.38}$$

To decide the amount of time each stream or combination of streams is available when the different streams have different availabilities, the proportion of the availability of the relevant stream or combination of streams to the availability of all the streams is computed. When one stream is available, these proportions are

$$\begin{aligned} s_1 &= \frac{0.988}{0.988 + 0.99 + 0.997} = 0.332 \\ s_2 &= \frac{0.99}{0.988 + 0.99 + 0.997} = 0.333 \\ s_3 &= \frac{0.997}{0.988 + 0.99 + 0.997} = 0.335 \end{aligned} \tag{3.39}$$

When two streams are working, the proportions are

$$\begin{aligned} s_{1,2} &= \frac{0.988 \cdot 0.99}{0.988 \cdot 0.99 + 0.99 \cdot 0.997 + 0.988 \cdot 0.997} = 0.3315 \\ s_{2,3} &= \frac{0.99 \cdot 0.997}{0.988 \cdot 0.99 + 0.99 \cdot 0.997 + 0.988 \cdot 0.997} = 0.3345 \\ s_{1,3} &= \frac{0.988 \cdot 0.997}{0.988 \cdot 0.99 + 0.99 \cdot 0.997 + 0.988 \cdot 0.997} = 0.334 \end{aligned} \tag{3.40}$$

The availability of the third subsystem, which consists of a single component is given the data table 3.1. By calculating these numbers together, one gets the result given in table 3.9. For example, 40% flow will be available if the first and third subsystem is functioning, and only component number 2 out of the tree components of subsystem number 2. The equation is

$$\begin{aligned} A_{40} &= a_1 \cdot a_5 \cdot a_{1oo3-2oo3} \cdot s_1 \\ &= 0.994 \cdot 0.985 \cdot 3.883 \cdot 10^{-6} \cdot 0.332 \\ &= 1.328 \cdot 10^{-5} \end{aligned} \tag{3.41}$$

The same is done for all other possible flows.

Flow available	**Availability**
40	$1.328 \cdot 10^{-5}$
50	$1.34 \cdot 10^{-5}$
60	$1.332 \cdot 10^{-5}$
90	0.00802
100	0.971

Table 3.9: Availability of different amounts of flow through subsystem 1 to 3 of the example

When this is to be combined with the next parallel subsystem, one must look at each possible amount of flow from upstream and consider what will be available downstream. First, availability must be computed for the different streams of the subsystem. This is done as above for the second subsystem, keeping in mind that there are only two streams in this case. The results are

$$\begin{aligned} A_{2oo2} &= 0.979 \\ A_{1oo2-2oo2} &= 0.0209 \\ s_4 &= 0.498 \\ s_5 &= 0.502 \end{aligned} \tag{3.42}$$

Combined with the different amounts of flow available from upstream, the result for availability of different amount of flow becomes as described in table 3.10. When calculating these availabilities, it is important to remember that if 90% flow comes from upstream, it may result in 50%, 60% or 90% flow downstream, depending on which components are available in the subsystem.

The average production availability of the system is then

$$40 \cdot 1.3279 \cdot 10^{-5} + 50 \cdot 0.0102 + 60 \cdot 0.0103 + 90 \cdot 0.00785 + 100 \cdot 0.951 = \underline{\underline{96.935}} \tag{3.43}$$

These computations could have been done by employing the merging rules from the method above. When other methods are used to compute the availability of the system and the streams, they will be used.

Flow available	**Availability**
40	$1.3279 \cdot 10^{-5}$
50	0.0102
60	0.0103
90	0.00785
100	0.951

Table 3.10: Availability of different amounts of flow through the system of the example

The usual computation of the availability if one assumes that the first parallel has configuration 1oo3 and the second 1oo2, is

$$A = a_1(1-(1-a_2a_3)(1-a_3a_4)(1-a_2a_4))a_5(1-(1-a_6)(1-a_7)) = \underline{\underline{0.979}} \tag{3.44}$$

If this is used to compute the production availability by computing with the full flow, the estimate will be optimistic as it does not account for the times when not all streams are working, and consequently, the system cannot produce full flow.

If in contrast one assumes that all components must be operational for the system to function, one would ensure that full flow can pass through whenever the system is working. To compute this availability one must assume configurations 3oo3 and 2oo2 for respectively the first and second parallel. This gives an availability as for a series system:

$$A_{\text{full flow}} = a_1a_2a_3a_4a_5a_6a_7 = 0.935 \tag{3.45}$$

Multiplied with the full flow, one gets the result that the production availability is 93.5 percent. This is a conservative estimate as it only accounts for the time when full flow can pass for the system. The actual production availability will lie somewhere between these two bounds. For organisations there may be much money saved by having more accurate estimates.

3.5.2 Computation of variability

To compute variability, the methods presented in section 2.2.2 are applied. To compute the variability and estimate the variance, the derivative of the system function with respect to the input parameters are needed. As the system availability is a function of the component availabilities, and the component availabilities are functions of the input parameters, the derivation can be written as follows

$$\frac{\partial y}{\partial \alpha} = \frac{\partial y}{\partial x_i}\frac{\partial x_i}{\partial \alpha} \tag{3.46}$$

Where α is an input parameter. The derivation of the structure function with respect to the different component values are given by

$$
\begin{aligned}
\frac{\partial y}{\partial x_1} &= (1-(1-x_2)(1-x_3)(1-x_4))x_5(1-(1-x_6)(1-x_7)) \\
\frac{\partial y}{\partial x_2} &= x_1 x_5(x_6+x_7-x_6x_7)(1-x_3-x_4+x_3x_4) \\
\frac{\partial y}{\partial x_3} &= x_1 x_5(x_6+x_7-x_6x_7)(1-x_2-x_4+x_2x_4) \\
\frac{\partial y}{\partial x_4} &= x_1 x_5(x_6+x_7-x_6x_7)(1-x_2-x_3+x_2x_3) \\
\frac{\partial y}{\partial x_5} &= x_1(1-(1-x_2)(1-x_3)(1-x_4))(1-(1-x_6)(1-x_7)) \\
\frac{\partial y}{\partial x_6} &= x_1(1-(1-x_2)(1-x_3)(1-x_4))x_5(1-x_7) \\
\frac{\partial y}{\partial x_7} &= x_1(1-(1-x_2)(1-x_3)(1-x_4))x_5(1-x_6)
\end{aligned}
\tag{3.47}
$$

For the availability calculations given above, the input parameters were MTTF and MTTR. If we let α_i be the MTTF, and β_i the MTTR, the component availabilities which were put into the structure function are given by

$$a_i = \frac{\alpha_i}{\alpha_i + \beta_i} \tag{3.48}$$

The derivatives are given by

$$
\begin{aligned}
\frac{\partial a_i}{\partial \alpha_i} &= \frac{1}{\alpha_i+\beta_i} - \frac{\alpha_i}{(\alpha_i+\beta_i)^2} \\
\frac{\partial a_i}{\partial \beta_i} &= -\frac{\alpha_i}{(\alpha_i+\beta_i)^2}
\end{aligned}
\tag{3.49}
$$

The derivatives are to be evaluated at the expected values of the parameters. When values are inserted, the results are

$$\begin{aligned}
\frac{\partial y}{\partial x_1} &= 0.98490 \\
\frac{\partial y}{\partial x_2} &= 2.9370 \cdot 10^{-5} \\
\frac{\partial y}{\partial x_3} &= 3.5244 \cdot 10^{-5} \\
\frac{\partial y}{\partial x_4} &= 1.1748 \cdot 10^{-4} \\
\frac{\partial y}{\partial x_5} &= 0.99390 \\
\frac{\partial y}{\partial x_6} &= 0.0068536 \\
\frac{\partial y}{\partial x_7} &= 0.013707
\end{aligned} \tag{3.50}$$

and the derivatives of the component availabilities evaluated at the expected values are

$$\begin{aligned}
\frac{\partial a_1}{\partial \alpha_1} &= 1.1857 \cdot 10^{-5} \\
\frac{\partial a_2}{\partial \alpha_2} &= 1.8999 \cdot 10^{-5} \\
\frac{\partial a_3}{\partial \alpha_3} &= 2.4507 \cdot 10^{-5} \\
\frac{\partial a_4}{\partial \alpha_4} &= 4.0584 \cdot 10^{-6} \\
\frac{\partial a_5}{\partial \alpha_5} &= 2.4267 \cdot 10^{-5} \\
\frac{\partial a_6}{\partial \alpha_6} &= 2.7232 \cdot 10^{-5} \\
\frac{\partial a_7}{\partial \alpha_7} &= 1.0964 \cdot 10^{-5}
\end{aligned} \tag{3.51}$$

and for the derivatives with respect to the MTTR, the results are

$$\begin{aligned}
\frac{\partial a_1}{\partial \beta_1} &= -0.0019762 \\
\frac{\partial a_2}{\partial \beta_2} &= -0.0016284 \\
\frac{\partial a_3}{\partial \beta_3} &= -0.0024507 \\
\frac{\partial a_4}{\partial \beta_4} &= -0.0014204 \\
\frac{\partial a_5}{\partial \beta_5} &= -0.0016178 \\
\frac{\partial a_6}{\partial \beta_6} &= -0.0019452 \\
\frac{\partial a_7}{\partial \beta_7} &= -0.0016447
\end{aligned} \tag{3.52}$$

To find the variance of y, equation (2.11) is employed. As the components are assumed to be independent, only the variance terms of the equation are different from zero, and the equation becomes

$$\mathrm{Var}(y) = \sum_{i=1}^{n} \mathrm{Var}(\alpha_i) \left[\frac{\partial y}{\partial \alpha_i}\right]^2_{X^0} + \sum_{i=1}^{n} \mathrm{Var}(\beta_i) \left[\frac{\partial y}{\partial \beta_i}\right]^2_{X^0} \tag{3.53}$$

The variances of the parameters are given in table 3.11. When the values are put into the equation above, the result is

$$\mathrm{Var}(y) = 1.3881 \cdot 10^{-8} + 4.2216 \cdot 10^{-6} = \underline{4.235481 \cdot 10^{-6}} \tag{3.54}$$

This is a very small variance, and that is to be expected, as it is estimated from the variation in the input parameters. It shows that the uncertainty in the input parameters do not influence the result significantly unless the variation is large.

Computation of confidence interval is not obvious as the distribution of y is not a common distribution, but a scalar. If it is assumed that the errors are normally distributed, the 95% confidence interval becomes

$$[0.97899 - 1.96 \cdot 4.2355 \cdot 10^{-6}, 0.97899 + 1.96 \cdot 4,2355 \cdot 10^{-6}] = [0.97898, 0.97900] \tag{3.55}$$

3.6 Renewal processes and their application in reliability calculations

A renewal process is a counting process. The most common example of a counting process is the Poisson process, which describes the number of

Component	**MTTF**	Var(MTTF)	**MTTR**	Var(MTTR)
1	500	25	3	0.5
2	600	22	7	0.7
3	400	16	4	0.3
4	700	30	2	0.2
5	600	18	9	0.9
6	500	15	7	0.5
7	600	20	4	0.8

Table 3.11: The variances of the MTTR and MTTF for the example data

occurances of a specific situation in an interval, for example the number of customers arriving at a store during one hour. In the Poisson process, times of arrival are recorded, and the time between arrivals are iid and exponential

A number of statistics are of interest when considering a renewal process. Let the time between renewal number $i-1$ and i be denoted T_i. Then the time to the nth renewal is

$$S_n = \sum_{i=1}^{n} T_i \tag{3.56}$$

The distribution of S_n will of course depend on the distribution of the T_is, and is found by convolution, see [11], but as this is often time consuming, approximations are often used. Further, one may be interested in the number of renewals up to a specific time. This is denoted

$$N(t) = \max\{n : S_n \leq t\} \tag{3.57}$$

and the renewal function $W(t)$ and density $w(t) = W'(t)$, where

$$W(t) = E(N(t)) \tag{3.58}$$

$N(t)$ and $W(t)$ can be derived from S_n, see [11].

Renewal processes can be utilised in reliability theory when one considers a repairable component. The component will fail after a time, but as perfect repair is often assumed it is reasonable to assume that the time to failure will have the same distribution when repairs have been performed. If an exponential distribution for the time to failure is assumed and time to repair is considered to be neglectible, we have a Poisson process.

If time to repair cannot be neglected, one has what is called an alternating renewal processes, see [11]. Alternating renewal processes are systems where two processes succeed each other so that time to the first event is decided by the first process, time to the second event by the second process, time to the third event by the first process again, and so on. In this manner, time to failure can be modeled by one renewal process and time to repair by another. In this situation, a renewal is considered to happen when a component is returned to operating condition, that is,

$$T_i = U_i + D_i \tag{3.59}$$

where U_i is the uptime of the component, or time to failure, and D_i is the downtime or time to repair.

In reliability, quasi-renewal processes are also relevant. These make it possible to model systems which have increasing or decreasing MTTF, that is increasing or decreasing failure rate. It is thus not possible to use a quasi-renewal process if the time to failure is exponentially distributed. The quasi-renewal process is defined by

$$\begin{aligned}
T_1 &= Z_1 \\
T_2 &= \alpha Z_2 \\
T_3 &= \alpha^2 Z_3 \\
&\vdots \\
T_n &= \alpha^{n-1} Z_n
\end{aligned} \tag{3.60}$$

Where T_i are the successive times to failure and $Z_i, i = 1 \dots n$ are iid.

With $\alpha > 1$, one gets a process with increasing MTTF, while $\alpha < 1$ will give decreasing MTTF.

Let $f_i(t)$ denote the distribution of T_i. It can then be shown that, see [14],

$$\begin{aligned}
f_n(t) &= \alpha^{1-n} f_1(t) \\
F_n(t) &= F_1(\alpha^{1-n} t) \\
s_n(t) &= s_1(\alpha^{1-n} t) \\
r_n(t) &= \alpha^{1-n} r_1(\alpha^{1-n} t) \\
E(T_n) &= \alpha^{n-1} E(T_1) \\
\mathrm{Var}(T_n) &= \alpha^{2n-2} \mathrm{Var}(T_1)
\end{aligned} \tag{3.61}$$

Where s_n is the survival function and r_n is the failure rate.

In [15] and [14], results are derived for the quasi-renewal situation. Among other results, it is shown that if the distribution of T_1 is in one of the classes IFR, DFR, IFRA, DFRA, NBU or NWU, then the distribution of T_n will belong to the same class of distributions. This is a logical assumption for a repairable component. A failure and repair may make the component deteriorate, but as the same factors must be assumed to be present to cause failures, the shape of the distribution of time to failure should be the same.

As for renewal processes, one can also have alternating quasi-renewal processes. It is then assumed that the repair rate increases or decreases for each repair. One may of course assume that the repair rate stays the same by letting $\alpha = 1$.

3.7 Renewal processes and availability

The availability of systems where the life of the components are modeled as an alternating renewal processes can be calculated from the properties already stated. The result for the limiting availability is the same as the usual result, see [11]:

$$A = \lim_{t\to\infty} A(t) = \frac{E(U)}{E(U)+E(D)} = \frac{\text{MTTF}}{\text{MTTF}+\text{MTTR}} \tag{3.62}$$

provided it is assumed that MTTR equals MDT. Availability at a specific point in time can be calculated using Laplace transforms, and the result is that

$$A^*(s) = \frac{1-f_U^*(s)}{s(1-f_U^*\cdot f_D^*(s))} \tag{3.63}$$

gives the Laplace transform of the availability. In principle, any failure and repair distributions can be entered into this result to obtain an expression for the availability. In practice, it will sometimes be very time consuming.

For the case of exponential time to failure and repair, the result can be calculated relatively easily, as shown in [11]. Let the failure distribution be

$$f_U(t) = \lambda \mathrm{e}^{-\lambda t} \tag{3.64}$$

and let the repair distribution be

$$f_D(t) = \mu \mathrm{e}^{-\mu t} \tag{3.65}$$

The Laplace transforms of these are

$$\begin{aligned} f_U^*(s) &= \frac{\lambda}{\lambda+s} \\ f_D^*(s) &= \frac{\mu}{\mu+s} \end{aligned} \tag{3.66}$$

When putting this into (3.63) one gets

$$\begin{aligned} A^*(s) &= \frac{1-\frac{\lambda}{\lambda+s}}{s(1-\frac{\lambda}{\lambda+s}\cdot\frac{\mu}{\mu+s})} \\ &= \frac{\mu}{\lambda+\mu}\cdot\frac{1}{s} + \frac{\lambda}{\lambda+\mu}\cdot\frac{1}{s+(\lambda+\mu)} \end{aligned} \tag{3.67}$$

From tables of Laplace transforms one can find that

$$\mathcal{L}^{-1}\left[\frac{1}{s}\right] = 1 \tag{3.68}$$

and

$$\mathcal{L}^{-1}\left[\frac{1}{s-\alpha}\right] = \mathrm{e}^{\alpha t} \tag{3.69}$$

so when inverting $A^*(s)$ one gets

$$A(t) = \frac{\mu}{\lambda+\mu}\cdot 1 + \frac{\lambda}{\lambda+\mu}\mathrm{e}^{-(\lambda+\mu)t} \tag{3.70}$$

By this method, one is able to compute the time dependent availability of the system. To find the system availability, the expression above is inserted into the structure function of the system.

3.7.1 Example

The method above is used for the example system shown in figure 3.1 with data as shown in table 3.1. Table 3.12 shows the availability functions for the components in the example.

Component	λ	μ	$\lambda+\mu$	$\mathbf{a(t)}$
1	0.002	0.33	0.332	$0.99398 + 0.006041\mathrm{e}^{-0.332t}$
2	0.00167	0.143	0.14467	$0.98846 + 0.011544\mathrm{e}^{-0.14467t}$
3	0.0025	0.25	0.2525	$0.99010 + 0.0099010\mathrm{e}^{-0.2525t}$
4	0.00143	0.5	0.50143	$0.99715 + 0.0028518\mathrm{e}^{-0.50143t}$
5	0.00167	0.11	0.11167	$0.98505 + 0.014955\mathrm{e}^{-0.11167t}$
6	0.002	0.143	0.145	$0.98621 + 0.013793\mathrm{e}^{-0.145t}$
7	0.00167	0.25	0.25167	$0.99336 + 0.0066357\mathrm{e}^{-0.25167t}$

Table 3.12: The availability functions for the components in the example

Next, these functions must be inserted into the structure function to find the availability function for the whole system. However, it is probably easier to insert the relevant time before the numbers are put into the structure function, as one otherwise has a large amount of calculation to do. In this case, it is wanted to calculate the availability at time $t = 10000$. When this is inserted into the availability functions given in table 3.12, the results are

$$\begin{aligned}
a_1(10000) &= 0.99398\\
a_2(10000) &= 0.98846\\
a_3(10000) &= 0.99010\\
a_4(10000) &= 0.99715\\
a_5(10000) &= 0.98505\\
a_6(10000) &= 0.98621\\
a_7(10000) &= 0.99336
\end{aligned} \tag{3.71}$$

These numbers are of course approximate. It is likely that the availabilities have converged or are close to converging to the limiting availability at time $t = 10000$. When the numbers are inserted into the structure function of the system, the resulting availability is

$$A(10000) = \underline{\underline{0.97903}} \tag{3.72}$$

To compute the resulting production availability, the method described in section 3.5 is applied. The resulting probability distribution for the system is given in table 3.13

x	$\mathbf{P(X = x)}$
0	0.020969
40	$1.17072 \cdot 10^{-6}$
50	0.0098786
60	0.0099510
90	0.0077358
100	0.95147

Table 3.13: Probability distribution for the flow through the system calculated with a renewal process.

The production availability estimated from these numbers are:

$$\begin{aligned} &40 \cdot 1.17072 \cdot 10^{-6} + 50 \cdot 0.0098786 + 60 \cdot 0.009951 + 90 \cdot 0.0077358 + 100 \cdot 0.95147 \\ &= \underline{\underline{96.9343}} \end{aligned} \tag{3.73}$$

To compute confidence intervals for the availability, the variance is estimated by equation (2.11). The derivatives

$$\frac{\partial A}{\partial a_i} \tag{3.74}$$

are as shown in equation (3.47). The derivatives

$$\frac{\partial a_i}{\partial \alpha_i} \tag{3.75}$$

where α_i is an input parameter, are not the same. The derivatives in this case are

$$\begin{aligned} \frac{\partial a_i}{\partial \lambda_i} &= -\frac{\mu_i}{(\lambda_i+\mu_i)^2} + \left(\frac{1-t\lambda_i}{\lambda_i+\mu_i} - \frac{\lambda_i}{(\lambda_i+\mu_i)^2}\right)\mathrm{e}^{-(\lambda_i+\mu_i)t} \\ \frac{\partial a_i}{\partial \mu_i} &= \frac{1}{\lambda_i+\mu_i} - \frac{\mu_i}{(\lambda_i+\mu_i)^2} - \left(\frac{\lambda_i}{(\lambda_i+\mu_i)^2} + \frac{t\lambda_i}{\lambda_i+\mu_i}\right)\mathrm{e}^{-(\lambda_i+\mu_i)t} \end{aligned} \tag{3.76}$$

These derivatives are to be evaluated in the expected values of the parameters μ_i and λ_i. t is considered to be a constant, and is set to the value $t = 10000$ as this is the value used in the calculations of the availability. New values must also be calculated for the derivatives in equation (3.47).

The derivatives evaluated at the expected values are given in tables 3.14 and 3.15

Component	$\left[\frac{\partial a_i}{\partial \lambda_i}\right]_{X_0}$	$\left[\frac{\partial a_i}{\partial \mu_i}\right]_{X_0}$	$\left[\frac{\partial A}{\partial a_i}\right]_{X_0}$
1	$-2,9939033$	$0,0181449$	$0,98495948$
2	$-6,8324911$	$0,0797920$	$0,0000276233$
3	$-3,9211842$	$0,0392118$	$0,0000321993$
4	$-1,9886089$	$0,0056874$	$0,0001118503$
5	$-8,8210372$	$0,1339194$	$0,9938886620$
6	$-6,8014269$	$0,0951249$	$0,0065013547$
7	$-3,9470907$	$0,0263666$	$0,0135020604$

Table 3.14: The derivatives evaluated at the expected value of the parameters.

Component	$\left[\frac{\partial A}{\partial \lambda_i}\right]$	$\left[\frac{\partial A}{\partial \mu_i}\right]$
1	-2.9489	0.017872
2	$-1.8873 \cdot 10^{-4}$	$2.2041 \cdot 10^{-6}$
3	$-1.2626 \cdot 10^{-4}$	$1.2626 \cdot 10^{-6}$
4	$-2.2242 \cdot 10^{-4}$	$6.3614 \cdot 10^{-7}$
5	-8.7671	0.13310
6	-0.044219	$6.1845 \cdot 10^{-4}$
7	-0.053294	$3.5601 \cdot 10^{-4}$

Table 3.15: The derivatives evaluated at the expected values

The variance is calculated to be

$$\mathrm{Var}(y) = \sum_{i=1}^{n} \mathrm{Var}(\alpha_i) \left[\frac{\partial y}{\partial \alpha_i}\right]^2_{X^0} + \sum_{i=1}^{n} \mathrm{Var}(\beta_i) \left[\frac{\partial y}{\partial \beta_i}\right]^2_{X^0} \tag{3.77}$$

when numerical values are inserted, the resulting variance is $\mathrm{Var}(y) = 1601.02$ and the standard deviation is $\sigma = 40.01$. The confidence interval becomes very large due to the large derivatives, so large that it has no meaning. This is due to the values of some of the derivatives being quite large, as can be seen in table 3.15. In addition, the variances of the input parameters are not very small. The results show that it is very important to have as small variance as possible in the input parameters. It also shows that this method may not be the best for this kind of availability calculation.

3.8 Quasi-renewal processes

For quasi-renewal processes, results for availability of several models are shown in [15].

The model most similar to the case in question is the model described in [15, Section 4.2.3]. The assumptions made for this model are:

- A unit starts working at time 0
- Upon failure it is imperfectly repaired, in the sense that the next time to failure will be a fraction α of the previous time to failure
- Repair time increases with a factor β for each failure
- After the kth failure, the system is imperfectly maintained in the sense of the (p, q)-rule at times T, $2T$,...
- Failures between maintenance are imperfectly repaired in the sense that the next time to failure will be a fraction δ of the previous time to failure
- Repair time for failures between maintenance is assumed to be negligible

Of course, quasi-renewal processes are not meaningful with an exponential distribution as the exponential distribution is memoryless.

In [15] it is shown that for the above model, the asymptotic average availability is given by

$$A(T,k;\alpha,\beta,\lambda,p) = \frac{\frac{\mu(1-\alpha^{k-1})}{1-\alpha} + \frac{T}{p}}{\frac{1-\alpha^{k-1}}{1-\alpha} + \frac{\eta(1-\beta^{k-1})}{1-\beta} + \frac{T}{p} + w} \tag{3.78}$$

Where μ is expected life time and η is expected repair time of a new unit.

If maintenance is not taken into account, the availability is given by

$$A(k;\alpha,\beta) = \frac{\frac{\mu(1-\alpha^{k})}{1-\alpha}}{\frac{\mu(1-\alpha^{k})}{1-\alpha} + \frac{\eta(1-\beta^{k})}{1-\beta}} \tag{3.79}$$

This is found by using the principles for renewal-reward which can be found in [11].

According to [14], the assumptions for this formula are that the planning horizon is infinite, the failure rate is continuous, monotonously increasing and differentiable, the cumulative distribution of the first time to failure is absolutely continuous, $F(0) = 0$ and the unit begins to operate at time 0. These assumptions are true for the Weibull distribution with shape parameter 1.5.

For a such a distribution, the probability distribution is

$$f(t) = 1.5\lambda^{1.5}t^{0.5}\mathrm{e}^{-(\lambda t)^{1.5}} \tag{3.80}$$

and the cumulative distribution is

$$F(t) = 1 - \mathrm{e}^{-(\lambda t)^{1.5}} \tag{3.81}$$

It is easy to see that $F(0) = 0$, and it is infinitely differentiable as e^t is infinitely differentiable. The failure rate for the Weibull distribution is given as

$$\alpha\lambda^{\alpha}t^{\alpha-1} = 1.5\lambda^{1.5}t^{0.5} \tag{3.82}$$

which is a monotonously increasing function.

3.9 Example

For a Weibull distribution with shape parameter 1.5, the probability distribution is

$$f(t) = 1.5\lambda^{1.5}t^{0.5}\mathrm{e}^{-(\lambda t)^{1.5}} \tag{3.83}$$

and the cumulative distribution is

$$F(t) = 1 - \mathrm{e}^{-(\lambda t)^{1.5}} \tag{3.84}$$

It is easy to see that $F(0) = 0$, and $F(t)$ is infinitely differentiable as e^t is infinitely differentiable. The failure rate for the Weibull distribution is given as

$$\alpha\lambda^{\alpha}t^{\alpha-1} = 1.5\lambda^{1.5}t^{0.5} \tag{3.85}$$

which is a monotonously increasing function. The Weibull distribution accordingly satisfies the assumptions for the model above. The assumptions regarding the component, that the planning horizon is infinite and the components start working at time 0, can easily be assumed. An infinite planning horizon does not fully comply with the other examples, but as steady state availability is used with the other calculation methods, except for the simulation methods, it should not be too different. The problem is that the availability becomes a function of the number of failures, not of time. In the other examples, the availability has been calculated for $t = 10000$. Instead, $k = \frac{10000}{\mu+\lambda}$ will be used.

The data for the example is given in table 3.1. In addition, the factors α and β are needed. I will let $\alpha = 0.97$ and $\beta = 1.02$. By putting the numbers into equation (3.79). The results are

The structure function for the system is

$$X = x_1(1-(1-x_2)(1-x_3)(1-x_4))x_5(1-(1-x_6)(1-x_7)) \tag{3.86}$$

Component	k	Availability
No 1	20	0.99050
No 2	17	0.98297
No 3	25	0.98229
No 4	15	0.99597
No 5	17	0.97820
No 6	20	0.97812
No 7	17	0.99019

Table 3.16: Availability for the component computed by a quasi-renewal process

And when the numbers for the availability is put into this, the resulting availability for the system is

$$A_{\text{system}} = 0.968698 \tag{3.87}$$

To calculate the production availability, the method for going from known component availability to production availability is used. First the availability for different configurations of the subsystems is used. The results are shown in table 3.17

Subsystem	Configuration	Availability
1	1oo1	0.99050
2	1oo3	0.999999
2	2oo3	0.999984
2	3oo3	0.96167
3	1oo1	0.97820
4	1oo2	0.99979
4	2oo2	0.96852

Table 3.17: The availability for the subsystems in different configurations

Next, the ratio between the availabilities for the different possibilities for on line components when not all streams are working. For the second subsystem, the possibilities are when one stream are working

$$\begin{aligned}
P(\text{comp 2 working}) &= \frac{a_2}{a_2 + a_3 + a_4} = 0.331947 \\
P(\text{comp 3 working}) &= \frac{a_3}{a_2 + a_3 + a_4} = 0.331717 \\
P(\text{comp 4 working}) &= \frac{a_4}{a_2 + a_3 + a_4} = 0.336337
\end{aligned} \tag{3.88}$$

Subsystem	Streams available	Availability
1	1	0.99050
2	1	$1.5 \cdot 10^{-5}$
2	2	$3.8314 \cdot 10^{-2}$
2	3	0.96167
3	1	0.97820
4	1	$3.127 \cdot 10^{-2}$
4	2	0.96852

Table 3.18: The availability of the different number of streams for the subsystems.

When two streams are working, the pos

$$\begin{aligned} P(\text{comp 2 and 3 working}) &= \frac{a_2 a_3}{a_2 a_3 + a_3 a_4 + a_2 a_4} = 0.330344 \\ P(\text{comp 2 and 4 working}) &= \frac{a_2 a_4}{a_2 a_3 + a_3 a_4 + a_2 a_4} = 0.334944 \\ P(\text{comp 3 and 4 working}) &= \frac{a_3 a_4}{a_2 a_3 + a_3 a_4 + a_2 a_4} = 0.334712 \end{aligned} \tag{3.89}$$

For the forth subsystem, the ratios are as follows when one stream is working:

$$\begin{aligned} P(\text{comp 6 working}) &= \frac{a_6}{a_6 + a_7} = 0.496934 \\ P(\text{comp 7 working}) &= \frac{a_7}{a_6 + a_7} = 0.503066 \end{aligned} \tag{3.90}$$

From these numbers, the probability distributions for the different amounts of flow through the subsystems can be set up. For subsystems 1 and 3, the distributions are trivial, as they only have two possible states.

For subsystem number 2, the probability distribution is given in table 3.19, and for subsystem number 4, it is given in table 3.20. The merging rules described in chapter 3.3 are then used to obtain the results for the whole system. The resulting probability distribution for flow is given in table 3.21.

The expected production availability is then

$$40 \cdot 4.8234 \cdot 10^{-6} + 50 \cdot 0.015060 + 60 \cdot 0.015246 + 90 \cdot 0.012043 + 100 \cdot 0.92635 = \underline{\underline{95.3868}} \tag{3.91}$$

The variance due to uncertainty in the input parameters are calculated as in the other instances. Again, the derivatives of the component availabilities with respect to the input parameters are what needs to be recalculated. The input parameters of concern here are λ_i and μ_i. The derivatives of

x	$P(X = x)$
0	$1 \cdot 10^{-6}$
40	$4.9792 \cdot 10^{-6}$
50	$5.0451 \cdot 10^{-6}$
60	$4.9758 \cdot 10^{-6}$
90	$1.2833 \cdot 10^{-2}$
100	$1.2657 \cdot 10^{-2}$
110	$1.2824 \cdot 10^{-2}$
150	0.96167

Table 3.19: The probability distribution of the flow through the second subsystem.

x	$P(X = x)$
0	$2.1 \cdot 10^{-4}$
50	$1.5539 \cdot 10^{-2}$
60	$1.5731 \cdot 10^{-2}$
110	0.96852

Table 3.20: The probability distribution of the flow through the fourth subsystem.

x	$P(X = x)$
0	0.031297
40	$4.8234 \cdot 10^{-6}$
50	0.015060
60	0.015246
90	0.012043
100	0.92635

Table 3.21: The probability distribution for the flow through the system

interest are

$$\frac{\partial a_i}{\partial \lambda_i} = \frac{\frac{1-\alpha^k}{1-\alpha}}{\frac{\lambda_i(1-\alpha^k)}{1-\alpha} + \frac{\mu_i(1-\beta^k)}{1-\beta}} - \frac{\frac{\lambda_i(1-\alpha^k)^2}{(1-\alpha)^2}}{\left(\frac{\lambda_i(1-\alpha^k)}{1-\alpha} + \frac{\mu_i(1-\beta^k)}{1-\beta}\right)^2} \tag{3.92}$$

and

$$\frac{\partial a_i}{\partial \mu_i} = -\frac{\lambda_i \left(\frac{1-\alpha^k}{1-\alpha}\right)\left(\frac{1-\beta^k}{1-\beta}\right)}{\left(\frac{\lambda_i(1-\alpha^k)}{1-\alpha} + \frac{\mu_i(1-\beta^k)}{1-\beta}\right)^2} \tag{3.93}$$

The results when numbers are inserted are shown in tables 3.22 and 3.23

Component	$\left[\frac{\partial a_i}{\partial \lambda_i}\right]_{X_0}$	$\left[\frac{\partial a_i}{\partial \mu_i}\right]_{X_0}$	$\left[\frac{\partial A}{\partial a_i}\right]_{X_0}$
1	$1,8822521$	$-0,0114076$	$0,9779888477$
2	$4,6346487$	$-0,0541249$	$0,0000691373$
3	$2,1944249$	$-0,0219442$	$0,0000664827$
4	$1,4081286$	$-0,0040272$	$0,0002921609$
5	$5,9969422$	$-0,0910445$	$0,9902861927$
6	$4,3010465$	$-0,0601545$	$0,0095049671$
7	$2,6688100$	$-0,0178277$	$0,0211996616$

Table 3.22: The derivatives evaluated at the expected values

Component	$\left[\frac{\partial A}{\partial \lambda_i}\right]$	$\left[\frac{\partial A}{\partial \mu_i}\right]$
1	1.8409	-0.011157
2	$4.2042 \cdot 10^{-4}$	$-3.7420 \cdot 10^{-6}$
3	$1.4589 \cdot 10^{-4}$	$-1.4589 \cdot 10^{-6}$
4	$4.1140 \cdot 10^{-4}$	$-1.1766 \cdot 10^{-6}$
5	5.9387	-0.090161
6	0.040881	$-5.7177 \cdot 10^{-4}$
7	0.056579	$-3.7795 \cdot 10^{-4}$

Table 3.23: The derivatives evaluated at the expected values

The variance is given by the equation

$$\mathrm{Var}(y) = \sum_{i=1}^{n} \mathrm{Var}(\alpha_i) \left[\frac{\partial y}{\partial \alpha_i}\right]_{X^0}^2 + \sum_{i=1}^{n} \mathrm{Var}(\beta_i) \left[\frac{\partial y}{\partial \beta_i}\right]_{X^0}^2 \tag{3.94}$$

The variances are given in table 3.11. When numerical values are inserted, the variance becomes $\mathrm{Var}(y) = 719.64$ and the standard deviation is $\sigma = 26.83$. Also in this case the variance is large, resulting in a confidence interval of small interest, as it will include all possible values for the availability. Again, one must conclude that this method may not be preferrable if other options are available.

Chapter 4

Software programs for availability calculations

4.1 The Relex software

The Relex program consists of a number of different modules, made for calculations on reliability block diagrams (RBD), fault trees, doing FMECAs and a range of other methods. The modules of interest in this thesis are the reliability block module and the OpSim module. Both of these incorporates methods for calculating reliability, availability and related features for a system. In addition, the OpSim module contains methods for optimization of maintenance.

4.1.1 The principles of Relex

Each of the modules in Relex contains a different method of implementing the system. The components and data entered can then be accessed through the system file. Different implementations of the same system can be linked to avoid having to enter data several times. When a system is implemented, the model can be exported in several file formats, including text and Excel.

When doing calculations on a reliability block diagram (RBD), Relex uses a system to evaluate whether analytical solutions can be used or if simulation is required. If simulation is to be used, it uses Monte Carlo simulation. The program uses analytical calculations whenever possible. This is reasonable when one assumes that analytical calculations usually gives more accurate answers than simulation. Relex applies analytical methods unless they are unfeasible or the answer will be likely to be affected by round-off errors due to the complexity of the model, [1]. It is however possible to force the software to do Monte Carlo simulation.

According to the help section of the program, simulation must be used for reliability calculations if components in the RBD are repairable. For availability calculations, at least some measures may be calculated analytically also for repairable systems, as the example later in this chapter shows. Due to the constraints on analytical methods, they will be used relatively seldom.

When simulation is used, it is possible to calculate with repair resources, costs, switch mechanisms, conditional repair policies and other factors common in a real world system.

For the purposes of this thesis, the systems will be modeled as reliability block diagrams. This modeling methodology focuses on success of the system, that is, there being an operational path through the system. The method generally does not consider the flow through the system. In Relex it is possible to assign capacities to the blocks of the RBD, and get an estimate of the amount of flow through the system at different times.

Relex computes the different parameters of interest at a predetermined number of points during the interval of the calculation. The number of points is chosen by the user. This makes it possible to obtain results for the development of the system during the simulated time period.

4.1.2 Implementation of models in Relex RBD or OpSim

Models can be implemented directly as RBD diagrams, or one may enter the data into what is called the system sheet and later import the data to the RBD. All kinds of RBD structures can be implemented. Each component in the system must be modeled as a block in the diagram, and all data must be given per block. Redundancy can be modeled very easily if the redundant components are identical, by indicating the redundancy in the data for the block. If different components are in parallel, they must be implemented as separate blocks. The redundancy is then entered into the links between the blocks.

For each block one can specify failure and repair distributions, supported distributions are listed in table 4.1. If repair is to be considered in the calculations, this must be specified both when entering data for the component and when preparing calculations. A component can be repairable or replaceable or, if OpSim is in use, both. Needed repair recourses can be assigned and cost can be specified for the repair. Maintenance can also be specified, with many alternatives for how it is performed, for example one can choose whether a broken component is fixed on site or transported to a repair shop, and where spare parts are stored. Cost and time for transportation of parts can be accounted for.

It is in addition possible simulate what happens to components that are replaced or must be repaired; it is possible to specify repair on site or in a separate repair shop, with transportation time and cost of repairs. One can define both repair and replacement for every component, and imperfect repair can be modeled by the (p, q)-rule.

Generally, very complex maintenance strategies can be modeled, and one may come quite close to the strategies for a real life system.

Relex OpSim also contains the opportunity of optimizing the number of spare parts available and maintenance intervals with respect to cost or downtime of the system. If optimization is done, achieved availability can be calculated.

Distribution
Exponential
Lognormal
Normal
Rayleigh
Time independent
Weibull
Uniform
Constant time

Table 4.1: Probability distributions supported for modeling in Relex RBD and OpSim

Capacities can be calculated when simulation is used. The capacity and required capacity must be specified in each component. If the flow through a component falls below the required level, the component is regarded as failed. In addition, the type of network must be specified in the calculations dialog. Two types of network are available, electrical network and flow network. According to the help section of the program, the difference consists in how flow is distributed when there are components in parallel. In an electrical network, 100% flow is assigned to each parallel, while in a flow network, the flow is split so that if there are two parallel components, each is assigned 50% of the flow. It is not stated whether the capacities of the components in parallel are taken into consideration. If not, this can cause problems if one component has less or more capacity than is assigned to the parallel.

4.1.3 Calculations available in Relex RBD and OpSim

Calculations in Relex RBD are done either by analytical methods or by Monte Carlo simulation. An algorithm is applied to determine whether analytical methods are applicable, and if not simulation is used, see [2]. It is also possible to force the program to do simulation by marking a box in the calculation dialog. In the calculation dialog one determines which calculations are to be done and assigns parameters for simulation. One can choose how many system hours the calculation will consider and how many points the calculation results will be shown for. For simulation one decides how many replications are to be done and how many failures the system must experience to reach the steady state. The random number seed must also be specified. If two calculations are done with the same seed, the simulation results will be identical.

It is not possible to specify sensitivity for the calculation, but the results are shown with good sensitivity by default. Confidence level for calculation of confidence intervals can be specified. Confidence intervals can be calculated for reliability, availability, MTBF and MTTF.

If optimization is to be done, it is also specified in the calculation dialog. Optimization can be done for maintenance intervals and spare parts with respect to cost or downtime. Costs can be specified extensively when data is entered into the model, from cost of repairs, spare parts and other resources to general downtime costs and costs for transporting and storing components.

4.1.4 Availability calculations

According to the help section of the program, Relex applies the same definition as is used in NORSOK Z-016 for availability. Mobilisation time for resources is not accounted for in the availability measures.

Availability can be calculated in several ways in Relex OpSim. It can be analytical or simulated, and one can choose availability and operational availability, the difference being that operational availability takes into account the logistic delay time. In addition to the time dependent availability, Relex can calculate mean and steady state availability measures.

For a single unit with exponential failure time, the availability is calculated as

$$A(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} \mathrm{e}^{-(\lambda+\mu)t} \tag{4.1}$$

according to the help section of the program.

Mean availability is calculated according to the definition of interval or mission availability given in equation (2.3). The integrals involved are calculated by numerical integration. Steady state availability is calculated as the limit of the availability as time goes to infinity.

The analytical computation method is described in the help section of the program, and is as follows: For a series system the availability is computed as

$$A(t) = \prod_i A_i(t) \tag{4.2}$$

and for a parallel system as

$$A(t) = 1 - \prod_i (1 - A_i(t)) \tag{4.3}$$

These are basically the same equations as the ones used when applying the structure function to calculate availability, as described in chapter 3.1.

If the availability is calculated by simulation, the availability is calculated from the simulated failure and repair histories of the components.

4.1.5 Algorithm for Monte Carlo simulation

The simulation algorithm is presented in the help section of the program on two levels. The general method is:

1. The system is simulated the given number of times
2. The total number of system successes is counted
3. Calculate the expected reliability as system successes divided by system runs
4. Calculate other results

The individual simulation runs are done according to the following method

1. Find initial state of system
2. Generate random numbers from the specified probability distributions for each event that can occur in the system
3. Find next event
4. After every state change, repeat 1 and 2, with ages of components updated
5. Continue until the mission time or required number of failures has been reached
6. Find the vector of system state changes, use it to calculate the required measures for each simulation

It is also stated that the OpSim package performs simulation in a somewhat different manner, as it simulates the system only one time for each simulation iteration, until the required number of system failures to reach steady state has occur

Inverse transformation is used for drawing random numbers from probability distributions. For the normal and lognormal distributions that is not possible, but they can easily be drawn from in other manners, especially the Box-Cox algorithm. It is not stated whether this is the method employed by Relex.

4.1.6 Method for calculating steady state availability, failure frequency and MTBF

This method is described in [7], and in [8] it is stated that the method has been implemented in Relex. According to [7], the method applies to general failure distributions, and can be used with implicit decomposition methods. The algorithm is as follows, when ϕ_i denotes the failure rate and ψ_i denotes the repair rate of component i:

- Decompose the system into modules
- Find conditional failure and repair intensities for all components. They are always given by $\lambda_i^c = \frac{1}{\phi_i}$ and $\mu_i^c = \frac{1}{\psi_i}$
- Find the steady state availability of the components. It is given by $A_i = \frac{\phi_i}{\phi_i + \psi_i}$
- Find the steady state failure frequencies given by $\nu_i = \frac{1}{\phi_i + \psi_i}$
- Combine the found availabilities and failure frequencies to find the availability
- Continue until the top level is reached. Then, the MTBF is given by $\text{MTBF} = \frac{1}{\nu}$.

4.1.7 Capacity calculations

The capacity calculation is not the main focus of the Relex RBD and OpSim packages. The flow is calculated by recursively using the rules that for a series system, the capacity is the minimum of the capacity of the components, and for a parallel system, it is the sum of the capacity of the components. However, the algorithm might not be able to take into account ramp-up, variation in amount of flow, and so on. Relex does not include any method for maximizing the flow through the system.

4.1.8 Computation of confidence intervals

Confidence intervals are only computed for simulated results. For availability, the method of computing confidence intervals is somewhat unclear. According to the help section of the program, the computations are done as follows:

Let n be the number of simulation runs, and let n_s be the number of successful runs, that is, runs in which the system did not fail. The relevant probability, p, is then approximated by $\hat{p} = \frac{n_s}{n}$, and the upper and lower confidence limits are found by using the binomial equation, or by using the normal approximation to the normal equation. This method can be used for reliability and availability if repair is not taken into account. Otherwise, the confidence interval for availability assumingly is computed in some other manner.

It is important to remember that Relex considers the availability of the system as a whole, not of the components of the system. Downtime comes only when a cut set of the system has failed, and ends as soon as the system is up again.

For MTTF and MTBF, the confidence intervals are computed by estimating the mean and variance of the time to failure of the system from the times computed at each simulation.

4.1.9 Comparison of Relex and other reliability softwares

The article [5] compares the performance of the Relex RBD module with two other software packages designed for reliability evaluation, namely ARINC Raptor and Reliasoft BlockSim. These softwares are not a part of this project, but it was interesting to note that the results obtained from Relex, especially for availability, differed from the results obtained with the other software packages. In the article, four different models are implemented and simulated in each of the softwares. It seems the differences in the results grows larger as the model complexity increases.

The article does not aim to range the softwares, only to point out differences and caution the users of reliability softwares to the fact that it is very important to know the assumptions and methodologies applied by the software in use. It is possible that the differences in results between Relex and the two other softwares stem from the fact that Relex employs analytical methods as far as possible.

4.1.10 The example in Relex

The same example as was used before was implemented in the OpSim package. Figure 4.1 shows the diagram as it is rendered in Relex. Exponential failure and repair time was assumed, with data as shown in table 3.1. No costs, required resources or maintenance was implemented.

When left to decide for itself, the software used analytical methods for computing the availability. Then no confidence intervals were computed. The calculations were done for 10000 system hours, and shown at 11 points.

When simulation was forced, 10000 iterations were done, and 50 failures were required for the system to reach steady state.

When analytical methods were used, the results were as shown in table 4.2

And when the same model was simulated, the results became as shown in table 4.3

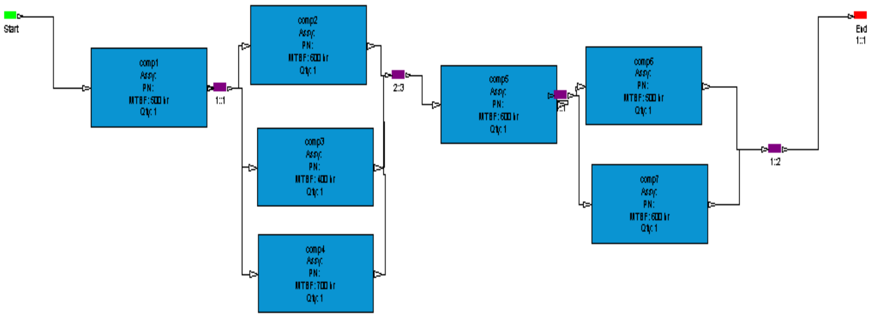

Figure 4.1: The model as shown when implemented in Relex

Measure	**Value**
MTTF	266.772885
Steady state availability	0.979256
Availability (at time 10000)	0.979256
Mean availability (at time 10000)	0.979271
Failure frequency (at time 10000)	3626.067796
Capacity (at time 10000)	95.94
Mean capacity	96.140747

Table 4.2: The results obtained with analytical calculations performed with Relex OpSim

Measure	**Value**
MTTF	275.451966
Steady state availability	0.979206
Availability (at time 10000)	0.9808
Availability LCL	0.97811
Availability UCL	0.98349
Mean availability (at time 10000)	0.979207
Failure frequency (at time 10000)	3636.1
Capacity (at time 10000)	96.328333
Mean capacity	96.140972

Table 4.3: The results obtained with Monte Carlo simulation performed with Relex OpSim

As can be seen, the steady state availability is very similar, and the analytically calculated availability at time 10000 lies inside the confidence interval of the simulated availability. The mean availability is also very similar, as is the mean capacity. The MTTF and failure frequencies are however somewhat different.

To be able to estimate the production availability of the system, calculations were done with different configurations of the parallel subsystems. For the above calculations, the configurations were respectively 1oo3 and 1oo2. In table 4.4, all the different results for the steady state and mean availability are summed up. These numbers can be used to compute the difference in availability for different configurations, and this can again be used to calculate an approximation of the production availability.

1st parallel	2nd parallel	Method	Mean	Steady state
1oo3	1oo2	Analytical	0.979271	0.979256
1oo3	1oo2	Simulation	0.979207	0.979206
1oo3	2oo2	Analytical	0.959454	0.959428
1oo3	2oo2	Simulation	0.959487	0.959405
2oo3	1oo2	Analytical	0.9791	0.979085
2oo3	1oo2	Simulation	0.979128	0.979079
3oo3	1oo2	Analytical	0.955676	0.955649
3oo3	1oo2	Simulation	0.955714	0.95569

Table 4.4: Mean and steady state availability for different configurations of the parallel subsystems

To approximate the production availability, one must know how many streams are available. As described in chapter 3.5, the different possible configurations of the parallels in the systems give numbers for different number of streams. With configuration 3oo3, all streams in the parallel must be online for the system to be available. This availability gives the availability of 3 streams. Configuration 2oo3 gives the availability of 2 or 3 streams. By subtracting the availability of 3 streams, the availability of exactly 2 streams is found. In the same manner, the availability of exactly 1 stream is found by subtracting the availability with configuration 2oo3 from the availability with configuration 1oo3. The same can be done for the fourth subsystem. The differences in steady state availability for the first subsystem are, when the simulated results are used:

$$\begin{aligned} A_{1oo3} - A_{2oo3} &= 1.27 \cdot 10^{-4} \\ A_{2oo3} - A_{3oo3} &= 0.023389 \end{aligned} \tag{4.4}$$

Second subsystem:

$$A_{1oo2} - A_{2oo2} = 0.019801 \tag{4.5}$$

In the current situation, there are no calculations for the subsystems alone, only for the system as a whole. For all calculations for the second subsystem, the configuration for the fourth subsystem is set to be 1oo2. Thus all the time the system is available and the current configuration of the second subsystem is available is accounted for. As the subsystems are connected in series, the time when not all subsystems are online is not interesting. To find the time when only one stream is online in the fourth subsystems for each of the configuration of the second subsystem, it is assumed that the percentage of time with only one stream available is the same independent of the configuration of the second subsystem. As the components are assumed to be independent, this is a logical assumption. The percentage of time with only one stream available in the second subsystem is

$$\frac{A_{1oo2} - A_{2oo2}}{A_{1oo2}} \times 100\% = \frac{0.019801}{0.979206} \times 100\% = 2.02215\% \tag{4.6}$$

Thus we have availabilities listed in table 4.5

2nd subsystem	**4th subsystem**	**Availability**	**Capacity**
3	2	0.936365	100
3	1	0.019325	55
2	2	0.022916	100
2	1	$4.7296 \cdot 10^{-4}$	55
1	2	$1.2443 \cdot 10^{-4}$	50
1	1	$2.5681 \cdot 10^{-6}$	50

Table 4.5: The availability of different number of streams for the subsystems, with the average capacity for the relevant number of streams.

There are no calculations available for the availability of the individual components. If it is assumed that the average amount is let through when less than all the components are online, for the first subsystem we get that the average when only one stream is working is

$$\frac{50 + 60 + 40}{3} = 50 \tag{4.7}$$

and when two streams are working the average is

$$\frac{110 + 100 + 90}{3} = 100 \tag{4.8}$$

For the second parallel, the average amount of flow if only one stream is working is

$$\frac{50 + 60}{2} = 55 \tag{4.9}$$

The capacities for the different combinations of available streams are shown in table 4.5. The production availability can from these numbers be estimated to

$$
\begin{aligned}
&100\cdot(0.936365 + 0.022916) + 55 \cdot (0.019325 + 4.7296 \cdot 10^{-4}) + 50 \cdot (1.2443 \cdot 10^{-4} + 2.5681 \cdot 10^{-6}) \\
&\quad = \underline{\underline{97.0233}}
\end{aligned}
\tag{4.10}
$$

4.2 Description of the reliability software MIRIAM Regina

4.2.1 General principles

MIRIAM Regina is primarily developed for the calculation of production availability. The software combines a flow algorithm with Monte Carlo simulation, and makes it possible to simulate highly complex flow networks. The description of the software is based on [3].

A system to be simulated in MIRIAM is entered as a flow network with system stages and items. Each system stage can consist of several items. Flow is defined in the system stages, and reliability parameters in the items.

Maintenance stragegies can be simulated. Spares and repair crews can be defined and limited. Mobilization time for resources is also a parameter. It is however not possible to model a system where sometimes a repair is performed and sometimes the broken component is replaced with a new component. One can model imperfect repair with the (p, q)-rule.

4.2.2 Implementation of a system in MIRIAM Regina

A system is entered as a flow network consisting of system stages and items, and data on maintenance and failure are entered for these. In addition, it is possible to enter system wide objectives. Quality constraints for the flow can also be modeled. There are many possibilities for modeling a flow which varies during the lifetime of the system. Both calendar profiles, which give the flow during the simulated time, and seasonal profiles, which give a variation in flow over a given period of time to be repeated throughout the simulation, can be implemented. In addition, for each system stage, ramp up can be modeled, that is, it can be modeled that after down time, the flow through the system stage must increase gradualy.

It is possible to have several input and discharge points in the network. Calendar and seasonal profiles can be defined for the flow. It is important to note that the reference level for the production availability is defined by the required flow throuth the discharge points, and if the capacities throughout the network are unbalanced so that the discharge points do not recieve enough flow even when all components are working perfectly, the production availability will be less than 100% even if the system was working perfectly during the whole simulation.

When implementing a network in MIRIAM Regina, one first defines all system stages. Each system stage can contain several items, but the flow capacity is defined in the system stage. Failure and repair data is defined in the items. A component in a system to be modeled will typically be represented by an item, while a system stage will be a collection of related components with the same flow capacity.

It is also possible to define storage tanks in the system. These can be defined to compensate for lack of upstream flow during downtime. If a system has several discharge points, they can be defined to compensate for each other at times when some discharge points achieve less than the demanded flow.

As the flow through the system is the central object of modeling, there are many opportunities for specifying properties of the flow. Several flows can go through the same system. Flows can be split in system stages so that two flows go out where only one went in (for example, in a seperator gas and oil will be seperated into two different flows, and this can be modeled). Quality constraints can also be defined. Proportional flow rules makes it possible to define relative amounts of flow through parallel stages.

Maintenance can be defined for the system. Several maintenance programs can be defined, and system stages choosen to be members of the program.

For repair and maintenance, spare parts and other required resources can be defined, with mobilization time for the resources. The number of available resources can be limited. Maintenance and repair can be implemented to take place immediately or be postponed until a zero flow opportunity. A maximal deferal time must then be stated. However, MIRIAM has no way of determining the length of the zero flow opportunities in advance, so this may also cause downtime for the streams.

It is also possible to define objectives for the model. These are not absolute requirements, but goals to be achieved if possible. For example, it can be an objective that one specific discharge point should always recieve as much flow as required. The objectives are defined in a prioritized list. If objectives conflict, the objective highest on the list is prioritized. Another modeling feature is simulation rules, which can be used to model dependencies. For example, it can be a rule that faliure of one item causes failure of other items. If rules conflict during simulation, this may cause unwanted results, see [3].

When modeling a system in MIRIAM Regina, it is important to keep the system as simple as possible to keep the simulation time down. Every time a change happens in the system, the flow algorithm is called, and if there are many limitations and objectives in the system, the flow algorithm will be time consuming.

A useful feature in MIRIAM is the possibility of modeling auxilary systems, that is, systems which are not a part of the flow network, but on which items in the flow network depend. Examples can be electric system or water distribution. Items can be modeled to fail or have degraded operation when the required amount is not produced by the auxilary system.

4.2.3 The simulation algorithm

MIRIAM Regina uses next event Monte Carlo simulation. The principle is that first, time to failure is drawn for all components in the system. These are then ordered. When an event occures, the time to repair for the component is drawn, and so on. Upon each event, the flow algorithm is called to calculate the flow through the system. An event can in addition to a failure or a repair be maintenance, alteration of flow due to step up or variation of the profile flow entered into the system.

4.2.4 Calculation of production availability

The production availability of a model simulated in MIRIAM is calculated by recording the flow through the system during the simulation and comparing it to the flow gotten if the system had been fully operational at all times. It is important to note that this value is calculated from the flow proscribed to be wanted throuth each discharge point. If the value does not correspond to the flow it is possible to get through the network, the production availability will be smaller than 1 even if the system operates perfectly at all times. This is a point to be noted during implementation of a system. In all cases the accuracy of the result will depent on the implementation of the system being according to the system caracteristics.

4.2.5 Available calculations

The main calculation done by MIRIAM is the calculation of production availability. In addition to the availatility for the entire system, the availability for each boundary point and each flow type is given. The results for each simulation are available, and the averages. Confidence intervals are not computed automatically, but can easily be estimated from the results for the simulation runs.

It is important to note that if buffer storage units are incorporated into the model, the production availability computed by MIRIAM Regina is not strictly calculated according to the definition given in NORSOK Z-016, as this definition does not include buffer storage, see [3].

MIRIAM also computes on stream availability, deliverability and demand availability, where the reference level is the demanded amount of flow through the discharge points. The amount of time when more than 0%, 20%, 40%, 60%, 80% and 100% of system flow is available are also computed automatically for each boundary point. From these, % availability can easily be computed. Deliverability is computed in accordance with the definition in NORSOK Z-016, given that all possibilities for substitution from others and buffer storage are modeled. Otherwise, these aspects of deliverability must be considered separately for the measure to comply with the definition, see [3].

On stream availability is computed as the percentage of time the flow through the system is larger than 0. This measure can be compared to the availability calculations done with other methods.

In addition to the availability measures, it is possible to obtain numbers for most of the features modeled in the system, such as amount of spare parts used and to which extent rules have been fulfilled. A blaming algorithm is also available. It assigns blame for downtime to the different components. However, it can not fully be relied on as in some cases, the wrong component may be blamed for the downtime, see [3].

4.2.6 Uncertainty measures

MIRIAM Regina computes the standard deviation for the deliverability. As mentioned above, the deliverability will in many cases be identical to the production availability. To compute confidence intervals, this measure can be used. It is also possible to chose how many decimals shall be used in calculations, the default being 3. The results for each boundary point and each stream for every simulation iteration are available in MIRIAM, and confidence intervals for some other measures may be computed from these.

The main limitations of the software is in the time required for running simulations. This puts limits on many features of modeling, which would otherwise make it possible to simulate a model quite close to the real world system. For large systems, the assumptions must be extencively simplified and the number of simulations limited, and this can limit the accuracy of the results.

It is in MIRIAM possible to simulate some dependence between components, for example by letting failure in one component cause failure of others. Thus it is not always necessary to assume that components are independent, as it is in many other models. However, this posibility is limited as many such rules will make the simulation time excessive. When implementing a model, one must choose carefully which dependencies are important for the operation of the system.

4.2.7 Implementation of the example in MIRIAM Regina

The example used in previous sections was implemented in MIRIAM Regina. Figure 4.2 shows the flow diagram as it is shown in MIRIAM.

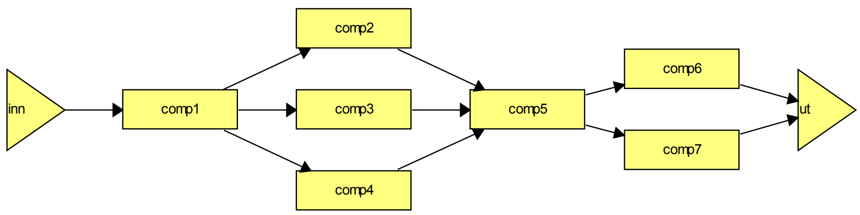

Figure 4.2: The model as shown when implemented in MIRIAM Regina

Data was as shown in table 3.1, and exponential distribution was assumed both for the failure and repair time.

Simulation was done over 10000 system hours. 1000 iterations were performed. The results are shown in tables 4.6 and 4.7.

Measure	Value
Production availability	96.95%
On stream availability	97.90%
Deliverability	96.95%
Demand availability	94.99%

Table 4.6: The results from the simulation in MIRIAM Regina

Percent flow	Percent time
0	2.10
0 – 20	0
20 – 40	0
40 – 60	0.65
60 – 80	1.32
80 – 100	0.93
100	94.99
> 100	0

Table 4.7: Percentage availability calculated by MIRIAM Regina

For the deliverability, the standard deviation is calculated automatically. In this case, it was $\sigma = 0.59\%$. For this model, deliverability is identical to production availability, as neither storage units nor substitution is modeled. For the production availability, the variance can also be calculated from the results from each simulation, by applying the standard formula for estimating variance,

$$\mathrm{Var}(y) = \frac{1}{n-1}\sum_{i=1}^{n}(y-\bar{y})^2 \tag{4.11}$$

The resulting estimated variance is

$$\mathrm{Var}(y) = 0.000034314 \tag{4.12}$$

which gives the standard deviation $\hat{\sigma} = 0.0058578$. Rounded to three significant ciphers, this is the same as was calculated automatically for deliverability.

A confidence interval for the production availability can be calculated based on the central limit theorem, as described in section 2.2.2. For the production availability, a 95% confidence interval is

$$\begin{aligned} &[\bar{y} - 1.96 \cdot \frac{\sigma}{\sqrt{n}}, \bar{y} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}] \\ &= [0.96947 - 1.96 \cdot \frac{0.0058578}{\sqrt{1000}}, 0.96947 + 1.96 \cdot \frac{0.0058578}{\sqrt{1000}}] \\ &= [0.96912, 0.96982] \end{aligned} \tag{4.13}$$

Chapter 5

Comparison of methods and results

5.1 Comparison of Relex and MIRIAM Regina

MIRIAM Regina and Relex are very different softwares. While Relex focuses on system failure or success, MIRIAM focuses on the flow through the system. While MIRIAM focuses almost exclusively on the production availability of the system, Relex provides a number of different results, including availability and an estimate of capacity. Which software is best depends entirely on the needs of the user. If production availability and the flow through the system is what is in focus, MIRIAM would be the best choice. Are other results wanted, Relex provides more opportunities for modeling and calculating different measures of the system.

It is not straightforward to compare the results of the softwares, as they focus on different aspects of a system. It will however be possible to compare the amount of time with no flow through the system in MIRIAM with the availability calculated in Relex.

Both softwares contain possibilities for including maintenance and spare parts in the models. An advantage in Relex is the possibility for optimizing maintenance intervals and spare part stocks.

Relex calculates a series of different statistics for the model: The MTBF, MTTF, availability and reliability and gives confidence intervals for all these. MIRIAM only gives the standard deviation for the production availability.

5.1.1 Implementation of models

Implementation of models is a simple task in both softwares, provided that the system is well understood in advance of implementation.

The principles of implementation are somewhat different. In MIRIAM, where the focus is on system flow, the system is modeled in two stages. First the flow network is implemented. The flow network consists of system stages in which the types of flow and capacity is decided. Each system stage can then have a number of items, which are the components of the system. Each item has separate failure and repair data. Maintenance is however implemented for system stages.

In Relex, each component must be modeled as a separate item in the system. All data must be entered for each component, including maintenance data.

5.1.2 Advantages and disadvantages of the two softwares

MIRIAM Regina provides a good software for modeling a flow network and computation of production availability and related measures. It is a large advantage that the results from each simulation can be obtained. The simulation methodology is straight forward and quite easy to understand. The flow algorithm is the basis for accurate calculation of the different measures, and a large part of the advantage of MIRIAM Regina. It does however make simulation time consuming, and limits the number of iterations feasible for large systems. The advantage of MIRIAM is that the computations made are intuitive and easily understood, and of course the complex flow algorithm which makes it possible to achieve accurate results for highly complex models.

Relex Architect makes it possible to compute a large number of different measures for the system. It is possible to compute the capacity of the system, but the algorithm is not as advanced as that of MIRIAM. The advantage of Relex is the large number of possible computations, and the possibility of adding data about cost to the model. Simulation is somewhat faster in Relex than in MIRIAM.

In both MIRIAM and Relex, it is possible to implement advanced repair and maintenance strategies for the model. Relex does contain some possibilities in this field which are not present in MIRIAM. For example, in Relex it is possible to define data for both repair and replacement of a component, and a replaced component can be repaired and become a spare part.

In Relex, it is also possible to do optimization of maintenance intervals or based on cost or system downtime. This is not possible in MIRIAM.

Both MIRIAM Regina and Relex Architect are good and reliable softwares for their uses. MIRIAM has a somewhat more limited field of applicability, but is better than Relex at those tasks it is made for. Relex gives a wider range of options for computation, and gives the possibility of applying analytical computations when feasible.

5.1.3 Uncertainty in the computations

When simulation is used, the uncertainty in both MIRIAM and Relex will primarily depend on the number of simulations done, in addition to the simplifications made when implementing the model. One simulation run does however not necessarily mean the same in MIRIAM and in Relex. In MIRIAM, a simulation run consists of simulating the entire period given in the simulation parameters, regardless of the performance of the system. In Relex, it depends on which module is applied. In OpSim, the simulation methodology resembles that of MIRIAM, in the RBD module, a simulation iteration will end when the system as a whole has failed, according to the help section of the program [2]. The computation of confidence intervals in Relex is not well documented except in a few special cases. It must be assumed that valid statistical methods are employed also when documentation is lacking.

In Relex, the output consists only of the end results with confidence intervals, but it is possible to have results computed at many points of the simulated time. In MIRIAM, one can obtain some results from each simulation iteration, but only mean measures over the entire simulated time. Standard deviation is computed for the deliverability, but confidence intervals must be calculated from this. One can also take advantage of availability of the results from each simulation iteration to compute variance.

Relex also provides the possibility of applying analytical computations when the system makes this possible. This is an advantage, as analytical computations do not depend on the number of simulations, which is often, at least for complex systems, decided at least in part by the time available. However, for complex systems analytical calculations are often unfeasible. Whether analytical calculation is possible also depends on which calculations are to be done. Simulation can be forced even in the cases where the software would choose analytical computation.

When it comes to the model assumptions, the programs are quite similar. By default, it is in both programs assumed that the components are independent. In MIRIAM there are several ways of modeling dependencies between components, but these will rapidly increase the simulation time. In Relex, there are no such features. This is an advantage for MIRIAM. In many ways, MIRIAM is more flexible when it comes to assumptions on the flow system modeled, while Relex is more flexible when modeling repair and maintenance. Again, the choice of software must depend on which features are considered most important. Also for repair and maintenance modeling, Relex and MIRIAM offers many similar features, but all in all there are more pos

All in all, which software is preferable depends on the computations to be done and which system features are considered most important. If system flow and production availability is the important issue, MIRIAM Regina would be the best software. If measures like cost and general system availability are the important points, Relex offers more opportunities.

5.2 Comparison of the analytical methods

Four analytical methods for computation of availability and production availability are presented in this thesis. Three of them are made for computation of availability, only one for production availability.

Which method should be chosen depends of course on what is to be calculated. The most flexible method is the method presented in section 3.3 for computation of production availability. The flexibility of this method is due to the application of Markov Chains for computing the probability distribution for the flow through the system. Markov chains are very flexible, and can account for almost any assumption about the system, as long as it is possible to find the transition matrix for the system.

Markov chains can also be emplyed to compute availability, see [11]. This is not considered in this thesis, but would have the same advantages as the computation of production availability by Markov chain methods.

The method for computing production availability from known availability is based on many assumptions which make the method less accurate than one might wish. This method should not be used if other methods for computing production availability are feasible.

For computing availability, three methods are presented in this thesis. These are the ordinary computation of availability by employing the definitions of availability together with the system function, presented in chapter 3.1, renewal methods presented in section 3.6 and quasi renewal methods, presented in section 3.8.

The difference in employing renewal and quasi renewal processes in stead of the definitions of availability mainly lies in the fact that when applying the definitions of availability, it is most often only possible to compute mean availability or limiting availability. For renewal processes, the result for the limiting availability is the same as the result obtained by employing the definitions, as shown in [11]. It is however possible to compute or at least estimate the renewal function in many cases. This will give the function of availability in time, as is computed for the example used in this thesis in section 3.7.1.

Quasi renewal processes are different in that they are made to compute results for processes which are improving or deteriorating. This is not possible for ordinary availability computations as done here, and neither for ordinary renewal processes. Deteriorating processes are a natural assumption in the real world, as it would be very strange if components in a system stayed as good as new throughout their lifetime when they are subject to stresses from their environment.

When it comes to effort of calculation, the easiest alternatives in most cases will be the ordinary calculation of availability, or the method from [9] for computation of production availability. Computation of production availability by employing the availability calculations done with other methods is less accurate and more complicated. If the model assumptions indicate that other methods should be used for availability calculations, the amount of computation is not infeasible. For large systems, any analytical computation would be time consuming.

The advantage of using analytical calculations over simulation is that the accuracy of the results does not depend on the available time for simulation. However, as the assumptions that have to be made about the system when analytical calculations are to be done can be very limiting, the divergence between the model and the real world system may become large.

5.3 Comparision of analytical and simulated results

Method	**Availability**	**Confidence interval**
Relex simulated	0.979206	$[0.97811, 0.98349]$
Relex analytical	0.979256	-
MIRIAM	0.9790	-
Ordinary	0.97899	$[0.97898, 0.97900]$
Prod. av	0.979	-
Renewal	0.97903	$[0, 1]$
Quasi-renewal	0.968698	$[0, 1]$

Table 5.1: Comparision of the results for availability from the different methods

The different computed values for the system availability are given in table 5.1. All the values for computed availability are the same when rounded to three decimal places, $A = 0.979$. The only exception is the quasi-renewal. This was expected, as this measure assumes that the system is deteriorating with each failure, while all the others assume a repair restores the system to good as new status.

The confidence intervals computed are based on different concepts. The confidence interval for the simulated availability from Relex is computed from the different results obtained in the simulation iterations. The confidence intervals are based on the uncertainty in the input parameters. What is shown by these confidence intervals is that for renewal and quasi renewal calculations, the uncertainty in the input parameters has much larger influence than for the other methods. For the ordinary method of analytical computation of availability, the confidence interwall becomes very narrow, which implies that uncertainty in the input parameters does not affect the result very much.

The values for production availability are given in table 5.2. These measures vary somewhat more than the values for the availability. As MIRIAM Regina and the method presented in section 3.3 are the only methods specifically meant for computing production availability, these measures are considered to be the most accurate. They are also very similar. When rounded to two decimal places, the results are the same, $A_{\text{prod}} = 96.95$. The methods based on the availability for the single components underestimate this measure somewhat, and the method based on the simulated availability results overestimate it slightly. Again, quasi-renewal gives a lower measure than the other methods, which is to be expected. It is worth notice that the capacity calculation done by Relex underestimates the production availability, the only method giving a smaller production availability is the calculations based on the quasi renewal process. This may be due to the way in which the flow is delegated through flow networks in the computations, see section 4.1.2.

The only method for which confidence intervals can be computed for production availability is the simulation done by MIRIAM. This is due to the manner in which the computations are done in the other cases. In Relex, one would assume that it would be possible to estimate a confidence interval for the capacity calculation if methods for such were implemented. As they are not, and the results for the different simulation iterations are not available in the software program, this cannot be done.

Propagation of uncertainty if also difficult for the analytical method used for computing production availability. For the availability measures employed to compute the probability distributions they might have been calculated, but to propagate these through the capacity calculations is not trivial.

It is worth noting that for all the analytical methods except quasi renewal, the estimated production availability lies within the confidence interval for the production availability computed by MIRIAM.

The system considered in this thesis was a very simple one. Due to this even crude methods must be expected to give relatively good results. For a more complicated system, the amount of work involved in computing the production availability from methods meant to compute availability will quickly become unmanagable. The results will probably deteriorate as the system compexity grows, as there will be more inaccuracies.

The difference between the methods for computation of production availability is the manner of computing the probability distribution for the flow. Markov chains give, as stated in [9], opportunity for implementing many features, by making the Markov chains more complex. The limitations of

Method	**Production availability**	**Confidence interval**
Relex simulated	97.0233	-
Relex capacity	96.14075	-
MIRIAM	96.95	$[0.96912, 0.96982]$
Prod. av from av	96.935	-
Prod. av	96.94533	-
Renewal	96.9343	-
Quasi-renewal	95.3868	-

Table 5.2: Comparision of the results for production availability from the different methods

this method are primarily due to the available time and resources for the calculation. For the other methods, many assumptions must be done about the model. This limits the accuracy of the results, as in many cases the assumptions will not fully comply with the conditions in the real world system.

Chapter 6

Conclusion

The concern of this thesis has been to present and compare different methods for computation of availability and production availability for a production system assumed to handle a flow of some kind of fluid. The same methods might be applicable if the flow instead consists of some manner of items being handled, but this must be considered in each individual case.

The thesis presents two software programs for reliability computations, MIRIAM Regina, which is focused on computation of production availability, and the RBD and OpSim modules of Relex Reliability Studio, which among other things can be used to compute the availability of a system. In addition, several different analytical methods for computation of availability and production availability have been presented.

It has been shown that the results computed for a simple example are quite similar for many of the methods presented. As the example is simple, it is to be expected that even crude methods can obtain good results. For the example used throughout this thesis, it is worth noting that many of the results computed are within the confidence intervals for the relevant measure computed by simulation done by the softwares used in the thesis. This supports the quality of the computed measures.

For availability computation, many methods can be employed, even methods primarily made for computing production availability if they give the amount of time with no flow through the system. This measure will be comparable to the availability except in cases where availability computations are done in a manner so that flow can pass through the system when the system is considered to be failed.

Availability is computed for the example system using several different methods. As the example system is very simple, the assumptions implied by the methods fit the system well for all the methods in this thesis. For the quasi renewal process, different assumptions must be made regarding distribution of time to failure and repair, but otherwise the assumptions are the same. The quasi renewal process is made to do computations on systems with an increasing or decreasing failure or repair rate. The results obtained from the quasi renewal process is thus quite different from the results obtained with other methods.

The main source of error when modeling a system, is the assumptions made during modeling. It is important when computations are made, and especially when results are interpreted, that the assumptions made are kept clear in mind. Calculations will only be accurate to the extent that the assumptions match the real world situation for the system. This is especially important to remember when results are computed with the help of software programs, where assumptions may not be clearly available unless the documentation for the software is considered in detail. It is thus important when using software programs, and especially when a person employs a software program for the first time, that time and effort is put into finding the assumptions made by the software.

The same caution must be employed when using analytical methods. The assumptions made are often more limiting for analytical computations than for simulation.

Other than the assumptions regarding the model, the main sources of uncertainty is uncertainty in the input data, and for simulation, the number of simulation iterations done. For some systems, the uncertainty due to propagation of error through the method can be estimated and a confidence interval made based on these computations. In a simulation setting, confidence intervals will be made based on the results obtained in the different simulation iterations. When doing simulation, propagation of uncertainty in the input data comes in addition to the uncertainty due to simulation, and methods exist for estimating the amount of uncertainty. However, these methods must be implemented in the software used prior to simulation. Otherwise, propagation of uncertainty cannot be estimated. As such methods are not implemented in MIRIAM or Relex, it is not possible to estimate the propagation of uncertainty in the simulations done with these softwares.

To conclude, when doing availability or production availability calculations, it is important to choose a method which enables the model to be as close to the real life system as possible. In most cases, this concern must be weighed against the available time and resources for the computations. The best choice of method will be the method which gives the most accuracy within these bounds. In many cases, especially for complex systems, this will be simulation. In many cases, analytical methods for availability calculation can be used to find a quick but crude estimate of the availability of a system. More complex analytical computations will primarily be of interest in situations where the alternative is to implement the software for simulation. If a software for sophisticated simulation of availability measures, analytical calculations will often be more time consuming than simulation, as both simulation and analytical calculations require a thorough understanding of the system on which calculations are to be done.

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